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Estimation of time-varying heat sources through inversion of a low order model built with the Modal Identification Method from in-situ temperature measurements

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ABSTRACT

An approach using an experimentally built low order model is proposed for the estimation of time-varying heat sources. In a first step, a low order dynamical system of equations, linking up temperatures at a set of specific points to heat sources strengths, is identified from experimental data using the Modal Identification Method. In a second step, the low order model is used to efficiently solve the transient inverse problem for the estimation of heat sources intensities from temperature measurements. The proposed approach is illustrated with an experimental set-up involving thermal diffusion with convective and radiative boundary conditions.

1. Introduction

In the framework of analysis and control of thermal systems, the knowledge of temperature evolutions at some monitoring points is often a basic requirement. When the number of monitoring points is large and/or when technical constraints prevent to place thermocouples or to use optical measurements, numerical simulation becomes a great tool to assess such quantities.

The knowledge of the involved heat transfer modes, geometry, boundary conditions (including heat exchange coefficients), thermophysical parameters, heat sources, is then needed to correctly model the system. If all these features are known, then the whole space and time-varying temperature field can be calculated.

When some of these features are unknown or not known accurately enough, and when some temperature data are available (other than those, unavailable, at desired monitoring points), then one has to deal with an inverse problem. Inverse problems are usually mathematically ill-posed and several regularization methods have been developed to ensure stable solutions (for an overview, see [1] for example).

The estimation of heat sources has been investigated by several authors. Concerning heat diffusion, the reader could refer to numerical works of Silva Neto and Özişik [2,3] where authors use the conjugate gradient method with the adjoint equation to solve a transient IHCP. In [2], both the source location and the timewise varying strength are unknown, whereas in [3] the space and time dependent strength of a volumetric heat source are sought for. Works by Le Niliot and Lefèvre [4–7] deal with the estimation of both time-varying strength and position of multiple static or moving sources and containing experimental tests. The authors use a formulation based on the Boundary Element Method and the estimation is performed in a sequential manner. They also provide several references to some works dealing with heat sources estimation, most of them being concerned with the estimation of time-varying strengths only. Inverse problems for the estimation of heat sources in natural convection have also been investigated, especially by Park and his co-workers who conducted several numerical studies. The conjugate gradient method has been used in [8] and [9], while a sequential approach based on Kalman Filtering has been developed in [10].

The usual way to model a thermal system is to build a numerical model based on a discrete form of the continuous equations governing heat transfer inside the domain under consideration. This leads to a system of algebraic equations (let us say N equations), which can be large due to zones with sharp temperature gradients and/or when 3D effects cannot be neglected.

Solving the inverse problem using such a large sized model, also called Detailed Model (DM), may be a quite long and difficult task.

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Nomenclature

$A(N,N)$	state matrix of DM
$B(N,p)$	command matrix of DM
C_p	heat capacity J kg^{-1} K^{-1}
$C_{obs}(q_o,N)$	observation matrix of DM
$F(n,n)$	diagonal matrix (modal form of diffusion term) of RM
$G\ (n,p)$	command matrix of RM
$H_o\ (q_o,n)$	output matrix of RM associated with vector $\widetilde{Y}_o$
$H\ (q,n)$	output matrix of RM associated with vector Y
h	convective exchange coefficient W m^{-2} K^{-1}
$M\ (N,N)$	matrix of eigenvectors of state matrix A
M	point in the space domain
N	order of DM (i.e. its number of equations)
n	order of RM (i.e. its number of equations)
nf	number of future time steps for function specification
p	dimension of input vector U
q_o	dimension of output vectors Y_o and $\widetilde{Y}_o$
q	dimension of vector Y associated with inversion data
Q	heat source strength W
t	time s
$T,\dot{T}(N)$	temperature vector, its derivative with respect to time K, K s^{-1}
$U\ (p)$	input vector
V	velocity m s^{-1}
$X, \dot{X}(n)$	RM state vector, its derivative with respect to time
$Y_o\ (q_o)$	DM output vector
$\widetilde{Y}_o(q_o)$	RM output vector
$Y\ (q)$	vector associated with data for inversion (part of $\widetilde{Y}_o$)
$Z(X)$	vector of nonlinearities in the reduced model

Abbreviations

DM	Detailed Model
RM	Reduced Model

Greek symbols

Γ	system boundary
Δt	time step s
ε	emissivity of the radiative surface
λ	thermal conductivity W m^{-1} K^{-1}
ρ	density kg m^{-3}
σ	Stefan–Boltzmann constant W m^{-2} K^{-4}
σ^m	standard deviation of measurement errors K
σ_U	mean quadratic discrepancy for inputs W
σ_Y	mean quadratic discrepancy for outputs K
Φ	heat flux density W m^{-2}
Ω	system domain or matrix applying vector Z (X)

Subscripts

k	time discretization
r	reduced

Superscripts

*	measured data for RM identification
me	measured data for inversion
T	transposition sign
-1	inverse of a matrix
id	related to RM identification
it	related to iterations for time steps of inverse problem

Model reduction methods aim to build low order models, that is, models involving a number of equations $n << N$. Reduced Models (RM) are able to reproduce the DM behavior with short computing time while preserving a satisfaying accuracy, and are very useful to solve inverse problems.

Among reduction techniques applicable to nonlinear problems, let us cite two of them. The Branch Eigenmodes Reduction Method (BERM) [11] has been used for building reduced models in the framework of heat sources strengths estimation. The BERM has been used to build low order models for the estimation of a single [12] and two [13] unknown heat source(s) from experimental data. The Proper Orthogonal Decomposition (POD) with a Galerkin projection [14], has been used in [15] to build a reduced model for the estimation of the time-varying intensity of a heat source in 2D nonlinear heat diffusion. This technique has also been used for inverse natural convection problems cited supra [9,10].

In these methods, the RM is obtained by computing modes of a specific spectral problem, and then by selecting or amalgaming the most dominant modes according to a particular criterion (temporal, energetic, ...).

In a different way, the Modal Identification Method (MIM) [16] is based on the identification of the reduced model parameters through the minimization of a squared residues functional built with the discrepancy between the responses of the system (outputs of a DM or in-situ measurements) on one hand and the outputs of the RM on the other hand, when specific input signals are applied. Reduced models built with the MIM have been used to solve efficiently several types of inverse problems with transient loads.

In [16], the MIM has been used in heat diffusion to build reduced models from a detailed one. These RMs have been used in [17] to solve an inverse boundary value problem for a single time-varying thermal input, using simulated data. Heat transfer was nonlinear due to the dependence of thermal conductivity on temperature, according to a linear relationship. That led to quadratic terms in the reduced model. In [18], the MIM has been used to identify low order models from an experiment. These models have then been used to simultaneously estimate five time-varying heat loads. Experimental data were also used for the inverse problem. The experimental apparatus was designed in order to ensure a linear relationship between thermal loads and output temperatures, especially with low radiative heat transfer. The limited temperature range also ensured constant thermophysical properties. In [19], the MIM has been used in turbulent forced convection to identify low order models from concentration measurements in a ventilated enclosure. These models have then been used to simultaneously estimate the time-varying intensities of two pollutant sources. Experimental data were used both for the model identification and the linear inverse forced convection problem.

The present paper constitutes a continuation of our works. It deals with the construction of reduced models for heat diffusion and convection with nonlinear boundary conditions, and the use of such reduced models for solving inverse problems. An experimental set-up is used to test the proposed approaches. It involves two time-varying heat sources, whose positions are supposed to be fixed, and temperatures measured by infrared camera and thermocouples.

The main differences between the proposed work and our previous ones are the following:

- In comparison with [16] and [17] dealing also with nonlinear heat diffusion, the present work deals with the estimation of two heat source intensities instead of one. The proposed inverse problem is trickier because each sensor is affected by both sources, and one has to discriminate them. Moreover, the low order models have been identified from temperature measurements recorded on the experiment, and no numerical model has been used. Experimental data have also been used for the