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The onset of buoyancy-driven convection in a horizontal fluid layer heated suddenly from below

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HIGHLIGHTS

- ▸ New approaches were applied to study the onset of transient Rayleigh–Bénard convection.
- ▸ Most dangerous initial disturbance was identified through the initial growth rate analysis.
- ▸ The neutral stability conditions were obtained under the relative stability criterion.
- ▸ Excellent agreement between the analytical analysis and the numerical simulation is obtained.
- ▸ Nonlinear numerical simulation results explained the previous experimental work quite well.

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ABSTRACT

The onset of buoyancy-driven convection in an initially quiescent fluid layer confined between the two infinite horizontal plates is analyzed theoretically. In the case of isothermal heating it is well known that a convective motion sets in for the Rayleigh number $Ra \geq 1708$. By using the linear stability theory, for $Ra \gg 1708$ the onset time of instantaneous instability t_c is analyzed in the self-similar coordinate. In the self-similar coordinate, the propagation theory based on the quasi-steady state approximation (QSSA) represents the eigenanalysis and the initial value approach without QSSA and therefore it seems to be a quite reasonable. It is assumed that, besides the theoretical t_c, there are two more characteristic times: (1) the theoretical time t_m which marks the maximum disturbance or fluctuation growth rate and (2) the experimental time scale t_D which marks the detection time of motion. By employing the numerical method under the single mode of instabilities, t_c and t_m are analyzed. It is interesting that t_c is the invariant but the predicted t_m-value is dependent upon the magnitude of initial conditions forced. It is shown for the isothermally heated system of a large Prandtl number, $t_m(\approx 4t_c)$ obtained by fitting some experimental t_D-value agrees well with the available experimental t_D-values for $Ra > 10^5$.
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1. Introduction

Conventional hydrodynamic stability theory determines the critical value of stability parameters such as the Rayleigh number and the Taylor number (Chandrasekhar, 1961). In most of the previous analyses the steady state basic flow and/or temperature fields are assumed. However, when an initially quiescent fluid layer is heated rapidly from below, the buoyancy-driven convection sets in at a certain time. The critical time $\boldsymbol{t_c}$ to mark the onset of convective motion becomes an important question and is not fully understood. The related hydrodynamical instability problem may be called an extension of the classical Rayleigh–Bénard problem.

The transient instability problem has been analyzed by using the quasi-steady state approximation (QSSA) (Morton, 1957) and its modification (Sun, 2012), amplification theory (Foster, 1965) and propagation theory (Kim et al., 2002), based on the linear stability theory. Under a somewhat different theoretical basis, the energy method (Homsy, 1973; Gumerman and Homsy, 1975; Kim et al., 2008) was employed to determine the lower bound to the onset time. Besides the above deterministic approaches, Jhavery and Homsy (Jhaveri and Homsy, 1980; Jhaveri and Homsy, 1982) and Kim and Kim (Kim and Kim, 1986) treated the transient stability problem statistically by introducing a random forcing function and solving the nonlinear equations numerically. Except for the propagation theory, most of the previous studies solved the stability problem in a layer of uniform thickness. Recently,

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Riaz et al. (2006) suggested the analytical, initial conditions and analyzed the stability in the semi-infinite boundary-layer coordinate. They called this model the dominant mode method which does not depend on the quasi-steady state assumption.

Here we will concentrate on the instability problem in an initially quiescent fluid layer heated isothermally from below from time $\boldsymbol{t}=0$. Although, the above theoretical methods have been tested for the present system, the following theoretical issues still remain: (1) What kind of the initial conditions are applied to analyze the stability problem as an initial value problem approach?; (2) What is the criterion on onset of convection?; (3) Are the obtained stability conditions independent of the solution methods? To answer the above questions, we derive the linear stability equation in the semi-infinite domain, and expand the temperature disturbance in terms of the time-dependent Hermite polynomials which are localized in the semi-infinite domain. The initial growth rate analysis shows that the first mode of the Hermite polynomial decays with time, while the rest of the spectrum decays more rapidly. We call this first mode the dominant mode disturbance and use an initial condition in the subsequent initial value problem approach (IVPA). Besides the IVPA, the spectral and the QSSA analyses are also conducted. The growth rate and the neutral stability condition under the relative stability criterion show excellent agreements between the various methods. These agreements seemed to be due to localized basis functions in the streamwise direction. In order to analyze the long-term evolution of the dominant mode disturbance predicted by the linear stability theory, we perform a nonlinear analysis using the conventional SIMPLE method. Considering the nonlinear growth of disturbances and comparing with the available experimental data, the present study will provide the bridges between the linear theory and the nonlinear theory, and between the theoretical predictions and the experimental observations.

2. Base system

The governing equations to describe the Boussinesq-flow in a horizontal fluid layer, where gravity points in the negative Z-direction, are

$$\nabla \mathbf{U}=0, \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\mathbf{U}\nabla\right)\mathbf{U}=-\frac{1}{\rho}\nabla P+\nu\nabla^2\mathbf{U}+\mathbf{k}\beta g T, \tag{2}$$

$$\frac{\partial T}{\partial t}+\mathbf{U}\nabla T=\alpha\nabla^2 T, \tag{3}$$

where $\mathbf{U}$ is the velocity vector, ρ is the density, P is the pressure, ν is the viscosity, T is the temperature, and α is the thermal diffusivity. The unit vector in the direction of the gravitational acceleration is $\mathbf{k}$. The schematic diagram of the basic system of pure conduction is shown in Fig. 1. The important parameter to describe the present system is the Prandtl number Pr and Rayleigh number Ra defined by

$$Pr=\frac{\nu}{\alpha} \text{ and } Ra=\frac{g\beta\Delta T d^3}{\alpha\nu},$$

where ΔT is the temperature difference across the fluid layer.

For a rapidly heated system, the stability problem becomes transient and the critical time t_c to mark the onset of buoyancy-driven instability has been analyzed by employing various methods mentioned in the introduction section. For this transient stability analysis we define a set of nondimensional time τ, vertical coordinate z and base temperature θ_0 by using the scale of time d^2/α, length d and temperature ΔT. Then the basic conduction state is represented in dimensionless form by

$$\frac{\partial\theta_0}{\partial\tau}=\frac{\partial^2\theta_0}{\partial z^2}, \tag{4}$$

with the following initial and boundary conditions,

$$\theta_0=0 \text{ at } \tau=0, \tag{5a}$$

$$\theta_0=1 \text{ at } z=0 \text{ and } \theta_0=0 \text{ at } z=1. \tag{5b}$$

The above equations can be solved by using the conventional separation of variables technique or the Laplace transform as follows:

$$\theta_0(\tau,z)=1-z-2\sum_{n=1}^{\infty}\frac{1}{\mu_n}\sin(\mu_n z)\exp(-\mu_n^2\tau), \tag{6a}$$

$$\theta_0^*(\tau,\zeta)=\sum_{n=0}^{\infty}\left\{erfc\left(\frac{\zeta}{2}+\frac{n}{\sqrt{\tau}}\right)-erfc\left(\frac{n+1}{\sqrt{\tau}}-\frac{\zeta}{2}\right)\right\}, \tag{6b}$$

where $\mu_n=n\pi$, $\zeta=z/\sqrt{\tau}$ and $\theta_0(\tau,z)=\theta_0^*(\tau,\zeta)$. Eq. (6b), which is based on the boundary-layer (τ,ζ) coordinates rather than the global (τ,z) ones, converges more rapidly than Eq. (6a) for the small times, i.e. $\tau \ll 1$. The evolution of the basic profiles of temperature with time is described in Fig. 2. With the above solutions the following relation can be derived using the chain rule:

$$\frac{\partial\theta_0}{\partial\tau}=\frac{\partial\theta_0^*}{\partial\tau}+\frac{\partial\theta_0^*}{\partial\zeta}\frac{\partial\zeta}{\partial\tau}=\frac{\partial\theta_0^*}{\partial\tau}-\frac{\zeta}{2\tau}\frac{\partial\theta_0^*}{\partial\zeta}. \tag{7}$$

The length $\delta_T(\tau)$ over which θ_0 is non-zero is the so-called 'thermal penetration depth' of the diffusive boundary layer. For the limiting case of $\delta_T(\propto\sqrt{\tau})\ll 1$, i.e. $\tau \le 0.01$, the domain can be considered semi-infinite in the positive z-direction, and the base

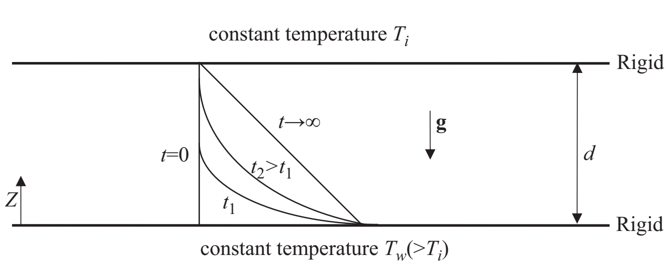

Fig. 1. Sketch of the basic conduction state considered here.

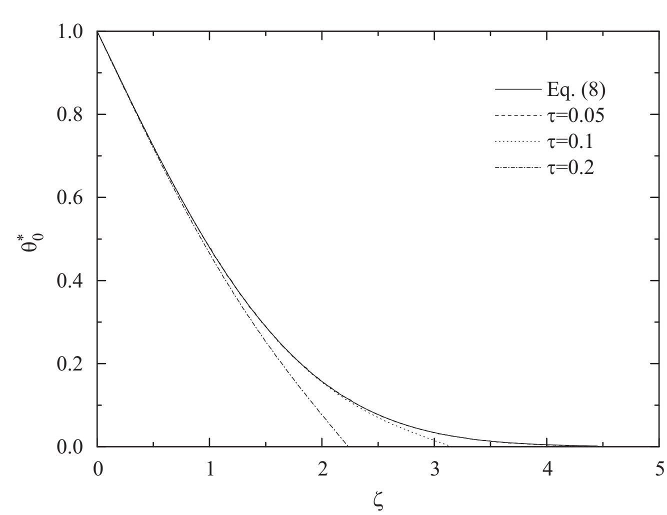

Fig. 2. Comparison of base concentration profiles. For the region of $\tau<0.05$, the similarity solution (8) gives good approximation to the exact solution (6).

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