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Unsteady conjugate mass transfer from a spherical drop in simple extensional creeping flow

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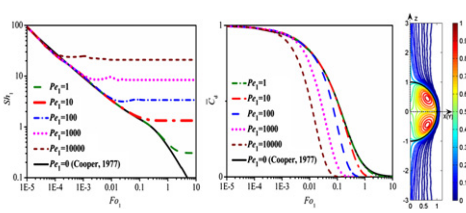
HIGHLIGHTS

- ▶ Conjugate mass/heat transfer from a drop in simple extensional flow is simulated.
- ▶ Interactive effects of *Pe*, viscosity ratio λ, diffusivity ratio K and distribution coefficient m are clarified.
- ▶ Conjugate transport approaches some limit cases if λ, K or m is far larger or less than unity.
- ▶ Flow direction has no influence on transport rates but affects concentration distributions.

GRAPHICAL ABSTRACT



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ABSTRACT

This paper presents the detailed numerical investigation on the unsteady conjugate mass transfer from a spherical drop immersed in a simple extensional flow. By making use of the known Stokes velocity field at small Reynolds numbers, a finite difference method with the control volume formulation is adopted to solve the convection-diffusion transport equations. The interactive effects of Peclet number, viscosity ratio, diffusivity ratio and distribution coefficient, as well as flow direction on the conjugate transport process are examined in terms of numerical simulation. Simulation results show that the conjugate mass transport is significantly influenced by these four parameters and it approaches some limit cases if viscosity ratio, diffusivity ratio and distribution coefficient become far larger or less than unity. The flow direction, no matter uniaxial extensional or biaxial extensional flow, has no influence on the total transport rate but affects a lot the distribution of solute concentration.

1. Introduction

The transport of mass and heat between a single particle and a surrounding fluid is very important in chemical, biochemical and other processing industries including solvent extraction, sedimentation of particles, spraying, atmospheric systems and so on, here the particle refers to gas bubble, liquid drop or solid particle. From a general point of view, the transport phenomena around a single dispersed body can be classified into three cases (Brauer, 1978; Clift et al., 1978):

(1) external problem, if the transport resistance inside the dispersed particle is negligible compared to that in the continuous phase;
(2) internal problem, if the transport resistance in the continuous phase is neglectable compared to that inside the dispersed particle;
(3) conjugate problem, when the transport resistance in both phases is comparable to each other.

The external and internal problems are usually regarded as the limit cases of the conjugate problem and have already been investigated extensively over the years. Most of the relevant literature concerned with the external and internal problems

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has been reviewed well by Brauer (1978), Clift et al. (1978), Sadhal et al. (1996) and Michaelides (2006).

In most cases, the mass/heat transfer from/to a particle should be regarded as a conjugate problem in which the transport processes both inside and outside the particle must be considered simultaneously. Because of its significance as a fundamental problem in multiphase processes, this subject has received a great deal of attention in the past years. For example, Philip (1964), Brown (1965), Konopliv and Sparrow (1970) and Cooper (1977) got separately the analytical solution for the transient heat transfer from a sphere in a stagnant fluid. In the uniform creeping flow regime, Brauer (1979) investigated firstly the conjugate heat transfer from a rigid sphere by means of theoretical–numerical methods and examined briefly the effects of Peclet number, thermal diffusivity ratio and volume heat capacity ratio; Abramzon and Borde (1980) numerically simulated the conjugate heat transfer from a liquid and a solid sphere and analyzed the influences of Peclet numbers on the transport process; Abramzon and Elata (1984) calculated the conjugate heat transfer from a solid sphere in which the temperature inside the particle was assumed to be uniform and the effects of Peclet number and volume heat capacity ratio were examined; Oliver and Chung (1986) extended the work of Abramzon and Borde (1980) by including the variable ratio of heat capacity, but the thermal diffusivity ratio was restricted to unity; Juncu and Mihail (1987) investigated numerically the effect of diffusivity ratio on the conjugate mass transfer from a drop in terms of numerical simulations; Juncu (1997) numerically solved the conjugate heat transfer from a rigid sphere to study the influence of thermal conductivity and volume heat capacity ratio, and he expanded this work subsequently (Juncu, 2001a) to the case of a fluid sphere, but the temperature inside the sphere was assumed uniform. In another article, Juncu (2001b) studied the effects of Henry number and diffusivity ratio on the conjugate mass transfer from both rigid and circulating fluid spheres, but the influences of Peclet number and viscosity ratio were not taken into consideration. Moreover, the conjugate problem from a drop in an electricity-driven creeping flow was studied analytically by Morrison (1977) and Chang et al. (1982) at high Peclet numbers and by Nguyen and Chung (1992) at low Peclet numbers. Beyond the creeping flow regime, the moderate Reynolds numbers domain was analytically solved by Chao (1969), Nguyen et al. (1993) and Uribe-Ramirez and Korchinsky (2000) and numerically simulated by Oliver and Chung (1990), Juncu (1998), Piarah et al. (2001), Waheed et al. (2002), Chen (2004) and Paschedag et al. (2005) successively, in which the influence of flow structure on the conjugate problem was investigated.

As mentioned above, most of the previous studies are focused on the systems in which the sphere moves at a constant velocity relative to the continuous phase. As is well known, the simple extensional flow is one of the basic liquid flows and frequently involved in many industrial processes such as fiber spinning, blow molding, melt squeezing and so on. In these processes, the mass/heat transfer occurs due to the concentration/temperature difference between some dispersed particles and the continuous liquid which will eventually affect the structural properties of products. In this sense, the investigation of mass/heat transfer from/to a small particle in extensional flow has both scientific and industrial significances, and hence has been carried out by several researchers. At very small or very high Peclet numbers, Batchelor (1979), Gupalo and Riazantsev (1972) and Favelukis (1998), Favelukis and Chiam (2003), Favelukis and Semiat (1996) studied the external problems separately for a single particle immersed in extensional creeping flow and derived analytical expressions of Nusselt number; Kurdyumov and Polyanin (1990) calculated numerically the external mass transfer from a sphere involving bubble, drop and solid particle in extensional creeping flow at finite Peclet numbers and proposed some approximate expressions for predicting the transfer rate. In our previous work (Zhang et al., in press), both the external and internal problems for a liquid drop and a solid particle in creeping flow were numerically solved for a wide range of Peclet numbers and the results obtained by Gupalo and Riazantsev (1972), Batchelor (1979) and Kurdyumov and Polyanin (1990) were compared and evaluated. With regard to the conjugate problem in extensional flow, only the work of Morrison (1981) is available, in which the unsteady heat transfer from a drop was solved analytically by means of the boundary layer theory that was approximately applicable only at high Peclet numbers.

Based on the above literature review, it is realized that although the conjugate mass/heat transfer from a particle in extensional flow is of great significance, the research work reported up to now is only restricted to the analytical solution applicable to high Peclet numbers, which is obviously not enough for comprehensive understanding and practical applications. On the other hand, even in the uniform flow that has been studied extensively, the conjugate problems are commonly examined only for some limiting values of Peclet number or part of influential parameters for the transport process. To our knowledge, no one has examined simultaneously all the four parameters including Peclet number, viscosity ratio, diffusivity ratio and volume heat capacity ratio (heat transfer) or distribution coefficient (mass transfer) on the conjugate transport processes at moderate Peclet numbers. In this work, the coupling convection-diffusion equations governing the internal and external transport processes are solved by using a finite volume numerical method for a drop in simple extensional creeping flow. We carry out a detailed parametric study to give a general description of the interactive effects of these four parameters as well as flow direction on the transport process.

2. Governing equations of mass transfer and scheme of numerical simulation

The particular geometry to be studied here corresponds to a spherical drop of radius a immersed in an ambient fluid of large extent. The velocity field far from the drop is assumed to be the simple extensional flow, which can be represented in the Cartesian coordinates by

$$\mathbf{u}_\infty = \mathbf{E}\cdot\mathbf{x},\ \ \mathbf{E} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} E \tag{1}$$

where $\mathbf{E}$ is the rate-of-strain tensor, $\mathbf{x}$ is the position vector and E is the strength of the extensional rate which is positive for uniaxial extension and negative for biaxial extension. The velocity field in the creeping flow regime has been well documented (Leal, 1992). In a spherical coordinate system (r,θ,φ) with the drop center at the origin, the dimensionless velocity scaled by $|E|a$ is (Zhang et al., in press)

$$\begin{aligned} u_{1r} &= \pm\left(1-\frac{3}{2}\sin^2\theta\right)\left(r-\frac{5\lambda+2}{2(\lambda+1)r^2}+\frac{3\lambda}{2(\lambda+1)r^4}\right) \\ u_{1\theta} &= \mp\frac{3}{2}\sin\theta\ \cos\theta\left(r\text{-}\frac{\lambda}{(\lambda+1)r^4}\right) \\ u_{1\varphi} &= 0 \end{aligned} \tag{2}$$

outside the drop, and

$$u_{2r} = \pm\left(1-\frac{3}{2}\sin^2\theta\right)\left(-\frac{3r}{2(\lambda+1)}+\frac{3r^3}{2(\lambda+1)}\right)$$