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# Vector lines and potentials for computational heat transfer visualisation

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ABSTRACT

Since the development of 3D computational heat transfer in the early 1970s, construction of the lines of vector fields has been a fundamental visualisation technique. In addition to the usual velocity lines, the lines of transport vectors for mass, energy and entropy can be especially relevant to heat and mass transfer. The use of scalar stream functions and vector potentials to ensure that these lines satisfy flux conservation is discussed in this paper by tracing the history of their use for streamline construction. A flux conservative method that uses an energy vector potential for constructing energy transport lines is described and its use for constructing 3D heat lines is demonstrated for natural convection in a cavity.
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## 1. Introduction

### 1.1. History

From the early beginnings of computational fluid dynamics and heat transfer (CFD/CHT), visualisation of the numerical solution fields has been almost as important as the solution processes themselves. The crux of the matter is that for 3D flows there is no suitable "whole field" method of flow visualisation as there is for 2D flow. Moreover, the most obvious method, that of constructing vector lines, is susceptible to errors arising from the forward stepping nature of the integration procedure and inaccurate interpolation of the numerical vector fields.

Prior to the development of 3D computational heat transfer, the stream function – vorticity method was very popular and the stream function trivially provided a whole field method of flow visualisation. Stream function contour lines are the lines the velocity or mass flux vector field and the difference between the stream function values of two lines is equal to the volume or mass flow between them.

Vector potentials or dual scalar stream functions can be used to represent 3D vector fields and early CHT solutions (Holst and Aziz [1], Mallinson and de Vahl Davis [2]) of 3D natural convection cavity flows used vector potential – vorticity methods. However there is no direct relationship between the vector potential and the lines of the velocity field that it represents. Visualisations of these solutions used planar maps and lines of the velocity vector fields. In practice issues associated with the prescription of boundary conditions for the vector potential and its lack of utility for visualisation meant that the vector potential – vorticity method has been almost completely bypassed in favour of primitive variable CHT methods.

Unlike the vector potential, dual stream functions do have a direct relationship with the lines of the vector field they represent. The directions of their gradients are normal to these lines and, in a manner similar to the 2D stream function, areas in stream function coordinates are proportional to the vector flow rates. Tantalising as this relationship might seem, the mathematical complexity of this relationship has precluded the development of a workable whole field visualisation strategy. Nevertheless, the search for an appropriate methodology has continued since the early 1970s when the challenges of 3D visualisation became apparent.

From the author's perspective, this search started with the realisation that potentials of some kind were necessary to ensure that interpolations of a discrete velocity field were mass conservative. As will be described in this paper a vector potential, trivially available as part of a vector potential – vorticity solution, can be used to provide mass conservative interpolations of a velocity or mass flux field. The desire to provide mass conservative interpolations from face centred staggered velocity fields used by PHOENICS (Spalding [3]) and its precursors led to the development of an algorithm, represented later in this paper, that used these data to integrate path lines analytically across each computational cell. This algorithm was embedded in GRAFFIC, (Mallinson [4]) which was first applied by Pollard and Spalding [5] to visualise flow in a tee-junction and then became the original 3D post processor for PHOENICS. It is still the author's algorithm of choice and the majority of visualisations presented in this paper have used it.

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**Nomenclature**

| | |
|---|---|
| **a** | arbitrary vector |
| **ds** | element of area ($m^2$) |
| $dv$ | element of volume ($m^3$) |
| $e$ | specific energy (J/kg) |
| **E** | total energy transport vector ($W/m^2$) |
| $f$ | arbitrary function, or one of the dual stream functions ($m^2/s$) |
| **f** | force per unit mass (N/kg) |
| $g$ | one of the dual stream functions ($m^2/s$) |
| $g_{ij}$ | metric tensor |
| **g** | gravitational acceleration vector ($m/s^2$) |
| $h$ | heat transfer coefficient ($W/m^2K$), mesh interval (m) |
| $k$ | thermal conductivity (W/mK) |
| $p$ | pressure ($N/m^2$) |
| $q^i$ | curvilinear coordinate (m) |
| $q'''$ | volumetric rate of heat generation ($W/m^3$) |
| **q** | heat flux vector ($W/m^2$) |
| $\dot{Q}$ | heat flow rate (W) |
| **r** | position vector (m) |
| $s$ | coordinate along a curve (m), specific entropy (J/kgK) |
| **S** | entropy transport vector ($W/m^2K$) |
| **m** | mass flux or mass transport vector ($=m_x\mathbf{i}+m_y\mathbf{j}+m_z\mathbf{k}$) ($kg/m^2s$) |
| $M_x$ | total mass flux through a cell in the $x$ direction (kg/s) |
| $Ra$ | Rayleigh number |
| $t$ | time (s) |
| **t** | tangent vector (m) |
| $T$ | temperature (K) |
| **v** | velocity vector ($=u\mathbf{i}+v\mathbf{j}+w\mathbf{k}$) (m/s) |
| $V$ | volume ($m^3$) |
| $\lambda$ | arbitrary function |
| $\rho$ | density ($kg/m^3$) |
| $\zeta$ | vorticity (/s) |
| $\zeta_m$ | mass vorticity ($kg/m^3s$) |
| $\Psi_m$ | 2D mass stream function (kg/ms) |
| $\mathbf{\Psi}$ | vector potential for velocity ($m^2/s$) |
| $\mathbf{\Psi}_e$ | vector potential for the energy transport vector (W/m) |
| $\mathbf{\Psi}_m$ | vector potential for the mass transport vector (kg/ms) |
| $\Phi$ | viscous dissipation function ($/s^2$) |
| $\theta$ | non dimensional Temperature |
| $\boldsymbol{\sigma}$ | deviatoric stress tensor ($kg/ms^2$) |
| $\boldsymbol{\tau}$ | stress tensor ($kg/ms^2$) |
| $\mu$ | dynamic viscosity (kg/ms) |

*Subscripts*

| | |
|---|---|
| $b$ | body (force) |
| $M$ | mechanical (energy) |
| $m$ | mass |
| ref | reference |
| $S$ | surface |
| $T$ | temperature or Thermal (energy) |
| $x,y,z$ | Cartesian coordinates |
| $\varphi$ | denotes energy transport vector that includes gravitational potential energy |

A major motivation for writing GRAFFIC was the need to have 3D graphics capability for understanding how dual stream functions might be used. GRAFFIC was written for direct view storage tube (DVST) technology which, although interactively inferior to the vector refresh technology that had already been used to construct visualisations of 3D convection, was orders of magnitude less expensive and provided a realistic graphics display technology for industrial CFD/CHT. Eventually GRAFFIC was used much more as a CFD/CHT post processor than it was for dual stream function research.

Ironically, the parabolic and partially parabolic methods developed and used by Brian Spalding's group at that time were, in fact, amenable to dual stream function representation. This was recognised by Brian Spalding who, in the 1990s, tacitly approved of Steven Beale's search for whole field dual stream functions (Beale [6] – chapter 7) described later in this paper.

### 1.2. Background of issues and problems

What does visualisation mean? In reality is not just graphics or colourful images; there are important conservation laws to represent. When users of CFD solvers and their visualisation methods produce falsely spiralling streamlines, observation clashes with intuition. Unfortunately even today, this is a too common reality. The majority of vector line methods used in commercial software exhibit such artefacts arising from well documented (e.g. Buning [7]) inaccuracies in either path integration or field interpolation.

The original motivation for research into improved algorithms for vector line construction is demonstrated by the natural convection example shown in Fig. 1. The spiral vector line is a streamline for this flow. The smoke visualisation provides clear evidence of the numerically predicted spiralling flow. A 10 cm by 10 cm by 2 cm cavity was heated and cooled by transparent side walls, filled with smoke and then left to settle before illuminating the vertical axial plane. The pattern forms naturally as the slowest moving air loses its smoke particles as they settle on the bottom the cavity thereby forming a continuous sheet of smoke free air. Although there are techniques for modelling smoke dispersion through cavity so that, in principle, the image in Fig. 1(b) could be simulated, a whole field method for constructing stream surfaces and also obviating false spirals is a necessary target if visualisation tools are to have the ability to rapidly define the flow as clearly as the smoke visualisation does.

As reviewed by Mallinson [8] there are several useful visualisation strategies that can be used for very complex 3D and 4D data fields. The present discussion will concentrate on the underlying technology for constructing vector lines. It will also consider the heat line methods proposed by Kimura and Bejan [9] that are of course relevant in the context of computational heat transfer. This paper therefore has two main threads. The first traces the development of research that seeks to find ways to construct the lines of vector fields while accurately maintaining the conservations laws that are represented by their divergence. The second thread considers heat or energy transport lines, their utility and how they may be constructed and used for 3D CFD/CHT fields.

## 2. Concepts and basic equations

### 2.1. Preliminaries

To set the mathematical context for the discussions in this paper it is worth reviewing some of the basic concepts associated with the vector fields of fluid dynamics and heat transfer.

An arbitrary vector field, denoted by **a**, may or may not have zero divergence. If it does it is described as being solenoidal. The divergence of **a** is related to the conservation of **a** over a control volume $V$ bounded by a closed surface $S$ by the Gauss divergence theorem

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