ARTICLE IN PRESS
JID: CNF [m5G;April 16, 2018;8:27]

Combustion and Flame 000 (2018) 1–12

Contents lists available at ScienceDirect

Combustion and Flame

journal homepage: www.elsevier.com/locate/combustflame

Nonlinear development of hydrodynamically-unstable flames in three-dimensional laminar flows

Advitya Patyal*, Moshe Matalon

Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA

ARTICLE INFO

Article history:
Received 23 October 2017
Revised 7 March 2018
Accepted 16 March 2018
Available online xxx

This paper is dedicated to Chung K. Law on the occasion of his 70th birthday, as a tribute to his distinguished contributions to flame instabilities.

Keywords:
Darrieus–Landau instability
Gas expansion
Cusp-like flames
Level-set methodology
Premixed flames

ABSTRACT

The hydrodynamic instability, which results from the large density variations between the fresh mixture and the hot combustion products, was discovered by Darrieus and Landau over seventy years ago, and has been named after its inventors. The instability, which prevents flames from being too flat, was thought to lead immediately to turbulent flames. Recent studies, initiated by weakly nonlinear analyses and extended by two-dimensional simulations suggest that this is not the case. It was established that the flame beyond the onset of instability, develops into a cusp-like structure pointing towards the burned gas region that propagates at a speed substantially larger than the laminar flame speed. In this work, we present for the first time a systematic study of the bifurcation phenomena in the more realistic *three-dimensional flow*. The computations are carried out within the context of the hydrodynamic theory where the flame is treated as a surface of density discontinuity separating burned gas from the fresh mixture, and propagates at a speed that depends on the local curvature and hydrodynamic strain rate. A low Mach-number Navier–Stokes solver modified by an appropriate source term is used to determine the flow field that results from the gas expansion and the flame is tracked using a level-set methodology with a surface parameterization method employed to accurately capture the local velocity and stretch rate. The numerical scheme is shown to recover the known exact solutions predicted in the weak gas expansion limit and corroborates the bifurcation results from linear stability analysis. The new conformations that evolve beyond the instability threshold have sharp crest pointing towards the burned gas with ridges along the troughs, and propagate nearly 40% faster than planar flames. Indeed, the appearance of sharp folds and creases, which are some manifestations of the Darrieus–Landau instability, have been observed on the surface of premixed flames in various laminar and turbulent settings.

1. Introduction

Combustion processes are characterized by considerable variations in temperature and density due to the large amount of heat released by chemical reactions. This significant heat release inevitably leads to gas expansion, inducing velocities that affect the flame propagation itself. Thermal expansion is the underlying cause of the hydrodynamic instability, first highlighted independently by Darrieus [1] and Landau [2], and commonly referred to as the Darrieus–Landau (DL) instability. Treating the flame as a surface of density discontinuity, with the flame speed along the flame front constant, Darrieus and Landau carried out a linear stability analyses and concluded that planar premixed flames are unconditionally unstable. This result seemed contradictory to the observation of stable flames in the laboratory reported nearly seventy years earlier by Mallard [3] and Mallard and Le Chatelier [4]. The growth rate, based on the Darrieus and Landau analyses, was found to depend linearly on the wavenumber implying that disturbances of short wavelength grow faster than those of long wavelength. Evidently, this result fails in the short wavelength regime where disturbances become comparable in size to the flame thickness and diffusion effects inside this zone could play a significant stabilization role. Subsequent work was carried by a number of investigators, the most notable results of which are due to Markstein [5] who also treated the flame as a surface of discontinuity, but assumed a dependence of the flame speed on the local curvature of the flame front through a phenomenological constant, which nowadays is referred to as the Markstein length. His analysis showed that for positive Markstein length (i.e., when the curvature is negative with respect to the burned gas), diffusion has indeed stabilizing influences on the flame propagation.

The most rigorous studies, based on asymptotic methods [6–8], were carried out in the early 1980s. Exploiting the disparity

---

* Corresponding author.
E-mail addresses: patyal2@illinois.edu, patyal.advitya@gmail.com (A. Patyal), matalon@illinois.edu (M. Matalon).

https://doi.org/10.1016/j.combustflame.2018.03.014

Please cite this article as: A. Patyal, M. Matalon, Nonlinear development of hydrodynamically-unstable flames in three-dimensional laminar flows, Combustion and Flame (2018), https://doi.org/10.1016/j.combustflame.2018.03.014

in length scales associated with hydrodynamic and diffusion effects, the role that heat, mass and viscous diffusion play on flame stability was quantified. This resulted in an explicit expression for the dependence of the Markstein length on the thermal expansion and Lewis number characterizing the mixture, and a dispersion relation that incorporates thermo-diffusive effects as a perturbation to the Darrieus–Landau (DL) growth rate. Both, Pelce and Clavin [6] and Frankel and Sivashinsky [7], linearized the governing equations about the planar flame solution and carried out a linear stability analysis to derive the aforementioned dispersion relation. The approach by Matalon and Matkowsky [8] is more comprehensive; exploiting the multi-scale nature of the problem they first derived a general model for treating the flame as a thin interior layer separating the fresh mixture from the burned gases, and then used it to examine the stability of planar flames. The dispersion relations obtained in all three studies[1] are identical, showing that diffusion effects act to stabilize the short wave disturbances in mixtures whose deficient component is the less mobile one, or equivalently in mixtures of positive Markstein lengths. It should be noted that the general nonlinear model derived by Matalon and Matkowsky [8], which has been cast in a coordinate-free form [9] and referred to as the *hydrodynamic model*, and its extension to account for temperature-dependent transport, arbitrary reaction orders and effects due to stoichiometry [10], allows studying flame propagation in arbitrary flow fields, including the stability of flames other than planar ones. For example, it was used to examine the stability of outgrowing spherical flames [11,12] and of strained flames [13]. The model is also appropriate for studying the nonlinear development that occurs beyond the instability threshold, which is the subject of the current paper.

Darrieus and Landau seemed to imply in their reported work that due to the instability premixed flames would necessarily be turbulent. Recent studies, however, suggest that this is not always the case. The first evidence was obtained from studies of the Michelson–Sivashinsky (MS) equation [14] that results from the hydrodynamic model by assuming weak thermal expansion. This equation describes the evolution of modestly-perturbed flames, and depends on a single parameter proportional to the domain size and inversely proportional to the Markstein length (scaled appropriately). Despite being nonlinear, the MS equation possesses exact solutions obtained by employing a pole decomposition technique [15–17]. The solutions of interest are the *coalescent pole* solutions, for which the flame acquires a cusp-like structure that propagates at a constant speed, and the members of this family are distinguished by the number of poles that contribute to the solution.The stability of these solutions in rectangular domains with periodic boundary conditions was established by Vaynblat and Matalon [18,19] who showed that for given conditions, there is always among the family of pole solutions a stable one, and the stable solution is the one that possesses the largest possible number of poles. When varying the domain size, for example, the solution exhibits a bifurcation phenomenon: in narrow domains, the long wavelength modes are irrelevant and, since the short wavelength disturbances are damped, the flat flame or zero-pole solution is the stable one. When increasing the domain size the flat flame loses its stability and the one-pole solution with a single peak and a higher propagation speed (than the planar flame) becomes the stable one. This solution becomes eventually unstable and the two-pole solution, with a deeper cusp and higher propagation speed becomes the stable one. This behavior continues as the domain size widens with the crest sharply increasing and the propagation speed tending towards a constant value. Numerical integration of the time-dependent MS equation confirms this scenario; starting with arbitrary initial conditions the solution after a sufficiently long time tends to the appropriate pole solution. This ultimately means that the consequence of the DL instability is the formation of cusp-like structures pointing towards the burned gas.

Although the MS equation provides valuable insight into the nonlinear development of the DL instability, the model is limited to weak thermal expansion that does not properly characterize combustion processes, which are typically associated with a temperature rise of approximately six-to-eight fold. This assumption also implies that the flame front is modestly perturbed and the flow weakly disturbed, which excludes the possible establishment of large-amplitude structures, and the generation of significant hydrodynamic strain rates that are known to have an effect on the flame propagation. For realistic values of thermal expansion, the nonlinear development of the flame front beyond the initial DL growth can be studied within the context of the hydrodynamic model, which permits large flame corrugations and is valid for arbitrary flow disturbances. The mathematical formulation consists of simultaneously solving the Navier–Stokes equations with different densities in the unburned and burned gases and tracking the flame front which propagates at a speed that depends locally on its curvature and the hydrodynamic strain. The flame speed is modulated by a Markstein length that lumps all the combustion characteristics, the heat release, the mixture properties and composition, and the fuel and oxidizer diffusion rates. The combustion and fluid dynamics are fully coupled; the flow is affected by the gas expansion that results from the heat release and the propagation is affected by the flow conditions through the flame speed.

Despite the simplification of the hydrodynamic model, its numerical implementation is nontrivial and challenging, requiring a robust hybrid Navier–Stokes/front-capturing scheme. Such a methodology was derived for two-dimensional flows, where the flame is practically a curve [20], and was used to examine the nonlinear development of the DL instability [21]. For simplicity, the small corrections to the Rankine-Hugoniot relations [8,10] on the order of the flame thickness were neglected, and the flame speed was assumed to depend only on curvature. In a following study [22] strain rate effects on the flame speed were included and the complete influence of flame stretch on the nonlinear propagation was investigated. These studies substantiate the conclusions drawn from the MS model, that beyond the instability threshold a single peak structure develops with wide curved troughs and a sharp crest pointing towards the burned gas, and that the amplitude and propagation speed of this new structure are significantly larger the stronger the instability becomes.

The main objective of the present work is to investigate the nonlinear flame development that results from the DL instability in a three-dimensional flow setting, with the flame an actual two-dimensional surface. Although three-dimensional numerical simulations were previously reported for model-type problems [23,24], this work constitutes the first systematic study of the bifurcation phenomena and the non-linear development of the instability in three-dimensions for *realistic conditions*. The results provide further insight into the topology and propagation speed of the steadily-propagating structures that ultimately evolve. Features of the induced flow caused by the instability and resulting from the gas expansion are discussed. In addressing these questions, nontrivial aspects of the numerical methodology had to be addressed; these include extending the level-set methodology for properly tracking a two-dimensional surface, introducing a parametrization of the flame surface and generalizing the immersed boundary method to compute local interfacial properties, such as gas velocity and flame stretch rate, which are needed for accurately determining the local flame speed.

---

[1] Although different notation and non-dimensionalization were used in [6–8], the expressions were verified to be identical.

Please cite this article as: A. Patyal, M. Matalon, Nonlinear development of hydrodynamically-unstable flames in three-dimensional laminar flows, Combustion and Flame (2018), https://doi.org/10.1016/j.combustflame.2018.03.014