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Numerical simulation for natural convection in vertical channels

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ABSTRACT

The investigation of laminar natural convection in vertical obstructed channels is conducted using an *h*-adaptive finite element algorithm. The adaptive model uses an L_2 norm based *a-posteriori* error estimator with a semi-implicit, time-stepping projection technique. The advection terms are treated using an explicit Adams Bashforth method while the diffusion terms are advanced by an implicit Euler scheme. By using the adaptive algorithm, mesh independent studies can be avoided. Results are obtained for thermal and flow patterns including average Nusselt numbers for different parameters (Rayleigh number, aspect ratio and locations of obstructions) in both smooth and obstructed channels.

1. Introduction

Natural convection flows in vertical channels with obstructions can be found in many engineering applications, e.g., heat exchangers, heat transfer in electric circuits and energy storage systems. Considerable experimental work as well as many numerical simulations has been carried on for many years in this area [1–9,15].

From a numerical aspect, much of the early numerical work stems from the techniques advocated by Spalding [11] and his students during the 1960s and early 1970s. Burch et al. [6] investigated the laminar natural convection between finitely conducting vertical plates by a finite-difference procedure. Said and Krane [5] investigated laminar natural convection flow of air in a vertical channel with a single obstruction numerically, using a set of finite element codes initially developed by Gartling [17]. Desrayaud and Fichera [7] simulated natural convection in a vertical isothermal channel with two rectangular ribs, symmetrically located on each wall. They used the SIMPLER algorithm [16], based on the initial pressure-based finite volume method technique described by Patankar [16], a former student of Spalding.

Generally when conducting natural convection simulation studies, a mesh independent study is typically needed for higher Ra values; these studies can become time consuming and costly. An alternative way to avoid this procedure is to apply adaptive meshing [12]. Many commercial CFD codes today now employ some form of low-level adaptive meshing. In this study, a locally *h*-adaptive mesh refinement algorithm is coupled with a Petrov–Galerkin finite element method (PFEM). Meshes are refined in regions where flow features change rapidly and coarsened where the flow properties are smooth and unvarying. The adaptation procedure is guided by an L_2 norm based *a posterior* error estimator. A semi-implicit, time-stepping projection technique is used for the flow solver. A more detailed description of the projection method is discussed in Ramaswamy et al. [10] and Wang and Pepper [13].

Simulation results for natural convection in a vertical channel are obtained first without any obstructions. A channel with one obstruction is then examined, followed by a channel with two obstructions. Thermal and flow patterns are obtained for different Rayleigh numbers, channel aspect ratios and obstruction locations. Results are compared with those obtained by Said and Krane [5] for natural convection in a vertical channel with one obstruction. Results for natural convection within a vertical channel with multiple obstructions are compared with experimental values obtained by Cruchaga and Celentano [2].

2. Governing equations and finite element formulation

The following non-dimensional relations are defined (non-dimensional terms are labeled with "*") for the governing equations of 2-D, incompressible fluid flows with natural convection effects (Boussinesq approximation and constant fluid properties are adopted):

$$x^* = \frac{x}{a},\ y^* = \frac{y}{L},\ r^* = \frac{r}{L},\ u^* = \frac{u}{\alpha/a},\ v^* = \frac{v}{\alpha/L},$$
$$P^* = \frac{P}{\rho\alpha^2/L^2},\ \theta^* = \frac{T - T_\infty}{T_w - T_\infty} \tag{1}$$

The corresponding non-dimensional conservation equations can be written as (after dropping the asterisks for convenience).

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Nomenclature

a	channel width
Ar	aspect ratio of the channel, b/L
e	error
$\bar{e}_{avg}$	average element error
L	channel height
L_1	distance to the obstruction on the left side wall from the entrance of the channel
L_2	distance to the obstruction on the right side wall from the entrance of the channel
m	total element number
N_i	shape function
P	pressure
Pr	Prandtl number
r	radius of obstruction
Ra	Rayleigh number
t	time
T	temperature
T_w	wall temperature
T_∞	ambient temperature
u,v	velocity components in x and y direction
Greek symbols	
$\overline{\alpha}$	Petrov–Galerkin weighting function
α	thermal diffusivity
β	thermal expansion coefficient
θ	non-dimensional temperature $(T-T_w)/(T_w-T_\infty)$
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
ξ_i	adaptation indicator
η	error index
$\bar{\eta}_{\max}$	maximum specified error

Conservation of mass

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

Conservation of momentum
x-direction:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\nabla p+\Pr\left(\frac{\partial^2 u}{\partial x^2}+Ar^2\frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

y-direction:

$$\frac{\partial v}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=-\nabla p+\Pr\left(\frac{\partial^2 v}{\partial x^2}+Ar^2\frac{\partial^2 v}{\partial y^2}\right)+Ra\Pr Ar\theta \tag{4}$$

Conservation of energy

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{\partial^2 \theta}{\partial x^2}+Ar^2\frac{\partial^2 \theta}{\partial y^2} \tag{5}$$

with the Rayleigh and Prandtl numbers defined as

$$Ra=\frac{g\beta(T_w-T_\infty)b^3}{\alpha\nu},\quad \Pr=\frac{\nu}{\alpha} \tag{6}$$

Bilinear quadrilateral elements are chosen in a Petrov–Galerkin weighted residual finite element approach (PFEM) to discretize the problem domains. The variables u, v and θ are replaced by using the trial functions

$$u(x,y,t)=\sum_{i=1}^{n}N_i(x,y)u_i(t) \tag{7}$$

$$v(x,y,t)=\sum_{i=1}^{n}N_i(x,y)v_i(t) \tag{8}$$

$$\theta(x,y,t)=\sum_{i=1}^{n}N_i(x,y)\theta_i(t) \tag{9}$$

The matrix equivalent form of the integral-based finite element equations can be expressed as

$$[M]\{\dot{V}\}+([K]+[A(V)])\{V\}=\{F_V\} \tag{10}$$

$$[M]\{\dot{\theta}\}+([K_\theta]+[A(V)])\{\theta\}=\{F_\theta\} \tag{11}$$

where a Petrov–Galerkin approximation is employed to assist in the discretization of the advection terms associated with velocity and temperature transport, i.e.,

$$W_i=N_i+\frac{\overline{\alpha}h_e}{2|V|}[V\nabla N_i] \tag{12}$$

where $\overline{\alpha}=\coth\frac{\gamma}{2}-\frac{2}{\gamma}$. A detailed description of the matrix relations and formalization of the numerical algorithm is described in Wang and Pepper [13].

3. Adaptation technology

The finite element method has been widely used in various engineering analysis areas for many decades. In particular, the use of adaptive techniques with the finite element method has resulted in accurate simulation results with overall reduced computational storage and solution times. The most popular CFD codes today employ some form of mesh adaptation.

In this study, an h-adaptive PFEM approach was selected to solve for fluid flow with convective heat transfer effects. An *a-posteriori* error estimator based on L_2 norm error calculation is adopted to guide the adaptation procedure [13].

A local element refinement indicator is defined to decide if a local refinement for an element is needed, i.e.

$$\xi_i=\frac{\|e\|_i}{\bar{e}_{avg}} \tag{13}$$

when $\xi_i>1$, the element is refined; when $\xi_i<1$ the element is coarsed. In an h-adaptive process, the new element size can be calculated by:

$$h_{new}=\frac{h_{old}}{\xi_i} \tag{14}$$

The entire adaptation procedure is shown in Fig. 1.

A more detailed discussion of the h-adaptive PFEM technique along with the error estimator as employed in this study is given in Pepper and Wang [14]. Additional work on the development and implementation of p- and hp-adaptation is described in Wang and Pepper [13].

4. Numerical simulations

4.1. Natural convection in a smooth channel

Natural convection in a smooth vertical channel was simulated as a first comparison study for an obstructed channel flow. Problem geometry and boundary conditions are shown in Fig. 2. The left and right walls are kept at high temperature, while the inlet is kept at a constant low temperature. The horizontal velocity is zero at the inlet. Both inlet and outlet pressures are also kept at zero.