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Non-linear dynamics of thermoacoustic eigen-mode interactions

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ABSTRACT

The natural eigen-modes of a combustion system are a strong function of its geometric design, temperature profile, and boundary conditions. Many practical systems have multiple, closely-spaced natural mode frequencies. While several prior studies have noted the nonlinear interactions that occur when multiple, linearly unstable modes are present, the nature of these interactions for closely-spaced modes has not been analytically treated. This paper treats this problem theoretically by utilizing a Galerkin expansion of the Euler equations, and the method of averaging to derive a set of amplitude equations for the modal amplitudes. In cases where the frequency spacing is large, many of the oscillatory terms have short time-scales and average out. However, in the case of closely-spaced modes, terms oscillating at the frequency difference between the closely-spaced frequencies correspond to long time-scales and, thus, remain after the averaging process. Results show that frequency spacing between the modes has significant impacts on limit-cycle amplitudes and their stability, even in the case of a static non-linearity. In addition, the conditions under which both modes can co-exist are a strong function of the frequency spacing. There are also certain conditions where one mode is completely suppressed by the other, even if both modes have positive linear growth rates; non-intuitively, the mode that is suppressed could be the one with the larger growth rate. The conditions under which one mode is suppressed are also a strong function of the frequency spacing parameter. Finally, for conditions in which one mode is suppressed, the limit cycle amplitude of the other mode is independent of the frequency spacing. Taken together these results show the difficulty in experimentally inferring the linear stability/instability of (non-dominant) combustor modes based upon "steady state" data taken during limit cycle conditions.

1. Introduction

Lean premixed combustion is the chosen mode of operation for low emissions gas turbines. However, this combustion regime is prone to thermoacoustic instabilities [1] where self-excited oscillations arise from coupling between unsteady combustion heat release and natural combustor acoustics. The large amplitude pressure oscillations can degrade hardware, increase maintenance time and costs and increase harmful emissions.

Increased attention to this phenomenon has resulted in several methods of controlling these instabilities and mitigating them in practical hardware [2–9]. The mathematical modeling of the system dynamics requires integrating models of the system's acoustic behavior and boundary conditions [10,11], the forced response of the fuel system [1] and the shear layers to acoustic excitation [12,13], and the unsteady heat release resulting from acoustic velocity, fuel, and vortical flow disturbances [14–16].

While these systems exhibit a variety of nonlinear behaviors, including limit-cycle oscillations, sub-critical bifurcations, and frequency locking, important simplifications into their dynamics can be obtained by recognizing that certain sub-elements can be treated as linear, while others are nonlinear. For example, Dowling [17] presented a model of nonlinear combustor thermoacoustics, utilizing a linear acoustic model, coupled to a nonlinear heat release model. The nonlinearity in heat release typically follows from the fact that the velocity perturbations (due to either acoustic or vortical disturbances) under limit-cycle oscillations are usually of the same order of magnitude as the mean velocity; i.e., $u'/u_0 \sim O(1)$. Unlike rocket combustion chambers where pressure/velocity amplitudes are of the same order of magnitude as the mean pressure/sound speed, gas turbine combustor pressure/velocity amplitudes are typically a few percent of the mean pressure/sound speed. Thus, the dominant non-linearity in the system is associated with unsteady heat release, coupling with linear acoustic perturbations [10,17–19].

If the acoustics are linear, they are governed by the linear wave equation; following Culick [20], we consider the following wave

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equation:

$$\nabla^2 p' - \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} = h \tag{1}$$

where

$$h = -\rho_0 \nabla \cdot \left[(\vec{u}_0 \cdot \nabla)\vec{u}' + \left(\vec{u}' \cdot \nabla\right)\vec{u}_0\right] + \frac{1}{c^2}\left[\Gamma(\nabla \cdot \vec{u}_0) + (\vec{u}_0 \cdot \nabla)\right]\frac{\partial p'}{\partial t} - \left(\frac{\Gamma - 1}{c^2}\right)\frac{\partial \dot{q}'}{\partial t} \tag{2}$$

Here, p' denotes the acoustic pressure, u' denotes the acoustic velocity, $\dot{q}'$ denotes the unsteady heat release rate, Γ is the ratio of specific heats and the quantities with subscript '0' are time-averaged values. Note that here, the density has been assumed to be homogeneous in each of the reactant and product regions as has been done in the past for acoustic analysis of such combustion applications. The effect of heat release is then applied as a jump/matching conditions across the flame location. The relationship between $\dot{q}'$ and p' for small amplitude perturbations is linear. Several such linear models have been established based on analytical models and experiments [14–16,21–24].

These linear models have successfully been used to understand and model the linear stability characteristics of combustors, such as the conditions under which self-excited oscillations occur. However, for larger amplitudes, non-linear effects such as saturation become important [14,25–28]. Linear models also fail when modeling the final oscillation amplitude. This is especially true in the case of subcritical instabilities wherein the zero solution is linearly stable but a stable finite amplitude limit-cycle still exists [1].

Despite this non-linearity in the heat release, if only one mode is linearly unstable, weakly-nonlinear describing function approaches have been used successfully to describe the system's limit-cycle response. In this describing function approach, an amplitude dependent term is incorporated along with the amplitude independent flame transfer function [10,14,17,25,26,29–31].

Most modeling studies to date have considered the nonlinear interactions of a single mode with itself. As multiple acoustic modes are present in a combustion system, the application of such analysis implicitly neglects the nonlinear interactions between these modes. Nonetheless, the occurrence of such nonlinear interactions has been established. For example, Lieuwen [32] presented a calculation with two linearly unstable modes. Purely linear considerations would suggest that both modes should be present with nonzero amplitudes. Moreover, a nonlinear analysis that only considered interactions of each mode with itself would similarly predict limit-cycle oscillations of each mode. *However, in the presence of nonlinear coupling through a saturating type nonlinearity (as is typically encountered for the response of the unsteady heat release to flow disturbances), only one mode has a significant amplitude and the other mode amplitude is effectively driven to zero.* Moreover, in practical systems and in experiments, often a single dominant unstable mode is observed. As noted from the calculation described above, this does not necessarily imply only a single unstable mode in the system. Rather, during the presence of multiple unstable modes in the combustor, one mode often suppresses the other and the dominant amplitude can also "hop" from one mode to another [14,33], such as shown by the data in Fig. 1.

Similarly, the time domain simulations by Stow and Dowling [34] clearly show how there are multiple solutions for the limit-cycle of the modes depending on the initial conditions, but they never found these modes to co-exist. Recent work [35] has shown that the lack of simultaneous non-zero amplitudes for multiple unstable modes is an inherent property of coupled oscillators when the modes have non-harmonically related frequencies. Even in the case of one unstable mode seen at a given time, the distinct transition between the modes can be observed for changes in operating conditions [14,33]. This transition region can depict both modes simultaneously while only one of them is finally observed.

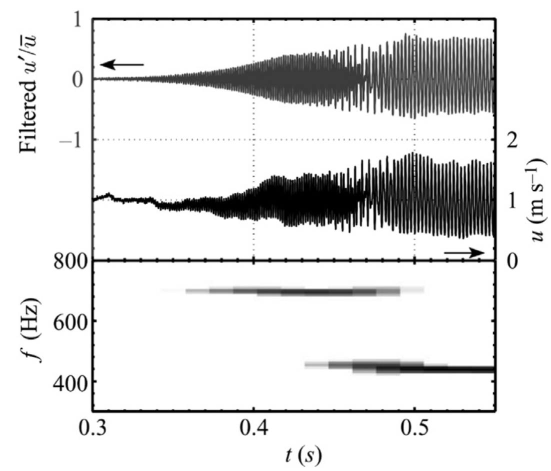

Fig. 1. Time-series data showing flow velocity measurements in a combustor with a fixed length. The upper box shows the measured time-series and the lower box shows the short time Fourier spectral density of the signal showing two bands of frequencies between which the dominant mode "hops". Figure reproduced from Noiray et al. [14].

Depending upon the system acoustics, the different potential acoustic modes can have quite different relationships in their frequency spacing. For an isothermal, constant area duct, the natural axial modes are either even ($f_n = nf_0/2$, where f_n denotes the nth acoustic mode frequency and n is an integer), or odd ($f_n = (2n-1)f_0/4$) multiples of the fundamental frequency f_0. In the presence of temperature gradients or area changes, the natural axial modes are not integer multiples [32], although they may be close. Similarly, radial or transverse duct modes are not integer multiples. In addition, there are instances where the natural modes may be closely spaced; i.e., $(f_1 - f_0)/f_0 \ll 1$. This can occur for axial modes with certain area/temperature profiles. Another instance is the pole splitting of modes caused by asymmetries (aerodynamic or geometrical) in the system. A very common instance where this occurs, is for transverse/spinning modes in round or annular ducts in the presence of mean azimuthal flow [36]. In these latter cases with low azimuthal flow Mach numbers, the slight difference in time of travel required for a spinning mode to propagate clockwise vs. counterclockwise induces this frequency shift.

In thermoacoustic systems, distinct peaks at non-harmonic frequencies have been shown in several experiments [37–39]. However, the pressure spectra do not indicate if the different oscillating states are simultaneously present or if the system "hops" or shares time between the different limit-cycles. However, detailed analysis of experimental data by researchers has shown the possibility of simultaneous limit-cycles at different frequencies [19]. Measurements have also indicated the possibility of these distinct frequencies being closely-spaced [5]. Figure 2 shows a pressure spectra obtained in a full-scale gas turbine combustion chamber. Two distinct peaks can be observed that are closely-spaced in frequency emphasizing the practical relevance of the study at hand. The normalized frequency spacing i.e. ratio of frequency spacing (Δ, defined later in Eq. (17)) to frequency of the first mode, Δ/ω_1, is approximately 0.3.

This paper particularly focuses on the nonlinear interactions of closely-spaced modes. The temporal dynamics of the modes are modeled using coupled oscillator equations which are then reduced to modal amplitude dynamics through the method of averaging. The problem of coupled oscillators was first treated for electrical circuits with a nonlinear element [40]. The coupling of thermoacoustic modes is analogous to these oscillators especially