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Dynamical response of a perfectly premixed flame and limit behavior for high power density systems

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ABSTRACT

The interaction between a perfectly premixed flame and an acoustic wave is studied in a 1D configuration. The mathematical framework to represent the flame dynamics is based on a recent work by Chen et al. that accounts for flame displacement in an acoustic field (Chen et al., 2016). An energetic approach is developed to estimate the source terms associated to flame/acoustic interactions. Three main contributions are identified: the *flame velocity response*, which corresponds to the flame response to upstream velocity perturbations (often modeled by an $n-\tau$ model), the *flame pressure response*, related to the link between the reactant consumption rate and the local density, and the *flame motion contribution*, related to the displacement of the heat release rate in an acoustic pressure field. The energetic contributions are evaluated theoretically using different formalisms to make connections with previous work. A focus is made on the limit case corresponding to high frequency/high power density (HF/HPD) configurations. This asymptotic condition is of interest in some practical applications, and a remarkable expression is obtained for the flame contribution in that case. The acoustic source term is expressed simply by the integral of the product between the mean heat release rate and the acoustic energy. A parametric study is carried out on a 1D duct of constant section incorporating a perfectly premixed flame. The spatial structure and frequency of the first mode (which is studied exclusively here) are obtained using a classical decomposition of pressure, velocity and entropy in plane waves and expressing jump and boundary conditions on the amplitudes. The flame energetic contributions are compared to one another and to the growth rate obtained using the classical wave method. The different methods are validated on a wide range of parameters. The particular behavior of systems featuring a high power density is presented.

1. Introduction

Combustion instabilities are an important technical concern when designing industrial systems which incorporate combustion chambers. They often result from the coupling between the unsteady heat released by the flame and the acoustic modes of the combustor. Understanding the phenomena driving the dynamics and response of a flame submitted to acoustic perturbations is of theoretical and practical interest.

Chen et al. [1] provided recently an interesting description of the dynamical behavior of lean premixed flames. They formulated the discontinuity between upstream and downstream of the flame using the Rankine–Hugoniot relationships, introducing the flame velocity in the laboratory frame of reference u_s'. This mathematical description provides a way to study two apparent problems that appear when the flame is considered immobile. The first one is that the mass flow rate is not conserved through an acoustic discontinuity. The second one is that velocity fluctuations upstream of the flame translates in significant entropy fluctuations downstream even for a perfectly premixed flame, which is not observed in experiments. Considering the fact that the flame position changes with time to adapt to upstream conditions allows to solve elegantly these conceptual issues. The impact of this displacement on the stability analysis of an actual system is not investigated in their study.

In parallel, an analysis of the impact of the motion of a flame on the acoustic source term is presented in [2]. An energetic approach is derived to estimate analytically the acoustic source term related exclusively to the flame displacement. One strong assumption of this study is that the flame follows *exactly* the acoustic velocity, which is expressed by the balance equation:

$$\frac{\partial \dot{q}}{\partial t} + \nabla \cdot \dot{q}\boldsymbol{u}_f' = 0 \qquad (1)$$

where $\dot{q}(\boldsymbol{x},t)$ is the local heat release rate and $\boldsymbol{u}_f'$ is the velocity of the flame in the laboratory frame of reference and chosen equal to the acoustic velocity. It is not true in general. This assumption

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was brought forth to isolate the contribution of the motion on the acoustic source term. In that case, it was proven that the contribution of the flame displacement to the acoustic balance equation was strictly positive and could become significant for systems with high power density. One reaches significant growth rate by rising the global heat release rate or by decreasing the dimensions of the system. This generally corresponds to an increase in the resonant frequencies of the system.

The first idea of the present work is to use the dynamical representation of the premixed flame provided in [1], apply it on a concrete case and estimate the different contributions, including the one due to the flame displacement discussed in [2]. As mentioned above, the displacement contribution becomes significant when the power density of the system increases. A particular focus is made to estimate the system behavior in the asymptotic condition corresponding to very high power density.

An energetic approach is chosen to account for the different contributions independently. There are two major contributors to the system stability: the coupling between flame and acoustics and the acoustic fluxes at the boundary conditions. In this study, this problem has been considered with much scrutiny because it appeared that some interesting features arise when the Mach number starts to grow. In addition, the formulation of the jump conditions by Chen et al. introduces an entropy term that is convected in the burnt gases. Therefore, an energetic formulation for the boundary contribution that includes entropy effect has been used [3].

The complex way premixed flames respond geometrically to acoustic perturbations is out of the scope of this paper. The G-Equation was used successfully in the past to model such phenomena [4–7]. Instead, the flame response is captured here using an $n-\tau$ model. The parameters n and τ are varied to study the influence on linear stability. Luzzato and Morgans [8] recently considered the effect of the flame motion on stability. The mathematical solution was obtained using the characteristic method in the temporal domain. This methodology provides interesting results, in particular because it gives access to non-linear features of the instability. Nevertheless the physical mechanisms which drive the impact on stability are not clearly identified and discussed. The study does not address the limit cycle for high-frequency modes and the role of entropy convection, which is investigated here. Using an energetic approach allows to provide a deeper physical understanding of why and in which context the implication of the motion is important.

The objectives of this article are the following:

- Use the dynamical description of the flame proposed by Chen et al. [1] to model the displacement contribution in an actual simple system.
- Introduce an energetic approach representing independently the different contributions from the flame and boundaries.
- Propose an asymptotic modeling of the flame contributions when the resonant frequencies and power density of the system increase.
- Present a parametric study to demonstrate the validity of the models and illustrate the influence of some key parameters: power density, Mach number, flame response, size of the cavity among others.

In Section 2, the formulation of the flame dynamics introduced in [1] is reviewed. Section 3 provides the various energetic contributions that will be compared in the parametric study. Section 3.1 introduces α_{Ray}, which accounts for the complete flame contribution, motion and flame response. Section 3.2 reminds the E-FAME model of [2]. In Section 3.3, the flame response is decomposed in two terms, a *flame velocity response* linked to velocity perturbations upstream of the flame and a *flame pressure response*. In Section 3.4, an asymptotic model is proposed for high frequency/high power density conditions. The boundary conditions contributions are estimated in Section 3.5. Section 4 describes the classical method to obtain the wave structure in the system. It also provides a reference growth rate which is used to compare to the energetic approach. Finally, Section 5 presents the results of the parametric study.

2. Formulation of the perfectly premixed flame problem

The idea of a moving discontinuity to represent a heat source has been introduced by Chu [9,10]. Subsequently, this idea was used in various studies ([11–13] among others). The particular feature of Chen et al. approach [1] is to link the entropy creation at the flame location and its velocity in the laboratory frame of reference. Using this formalism allows to provide meaningful explanations for apparent issues discussed previously in the literature, as already mentioned in the introduction. The entropy creation at the flame front becomes significant in some conditions and its interaction with the boundaries of the domain can have a dramatic effect on stability. The study of Chen et al. needs to be considered in this context. In this section, the main results of [1] are recalled and the formulation of the problem for the dynamical response of a premixed flame is put forward.

The Rankine–Hugoniot relations for a moving discontinuity are applied to the lean premixed flame using the notations presented in Fig. 1 case. Introducing u'_s, the velocity of the discontinuity in the laboratory frame of reference, it is possible to provide an expression for the jump conditions between upstream and downstream of the flame for the mean and the fluctuations of flow variables. In the following, the mean and fluctuating quantities (at first order) will be introduced: $f = \bar{f} + f'$. The conservation of mass, momentum and energy across a moving flame front of null mean velocity ($\bar{u}_s = 0$) reads for the mean values:

$$\begin{aligned} &\bar{\rho}_1\bar{u}_1 = \bar{\rho}_2\bar{u}_2,\\ &\bar{p}_1 = \bar{p}_2 + O(M^2),\\ &\bar{\dot{Q}} = \frac{\gamma}{\gamma-1}(\bar{u}_2\bar{p}_2 - \bar{u}_1\bar{p}_1) + O(M^2), \end{aligned} \tag{2}$$

and for the fluctuations:

$$\begin{aligned} &\frac{\rho'_2}{\bar{\rho}_2} - \frac{\rho'_1}{\bar{\rho}_1} + \frac{u'_2}{\bar{u}_2} - \frac{u'_1}{\bar{u}_1} = \frac{u'_s}{\bar{u}_1}\left(\frac{1}{\lambda} - 1\right),\\ &\frac{p'_1}{\bar{p}_1} = \frac{p'_2}{\bar{p}_2} + O(M^2),\\ &\frac{\dot{Q}'}{\bar{\dot{Q}}} = \frac{p'_1}{\bar{p}_1} + \frac{u'_2}{\bar{u}_2}\left(\frac{\lambda}{\lambda-1}\right) - \frac{u'_1}{\bar{u}_1}\left(\frac{1}{\lambda-1}\right) + O(M^2). \end{aligned} \tag{3}$$

where $\lambda = \bar{\theta}_2/\bar{\theta}_1$ is the ratio of mean temperature after and before the flame, u and p are velocity and pressure and $\dot{Q}$ refers to the heat release rate per cross-sectional area (W/m^2). Note that the jump condition is exact for the mass balance and of second order in Mach number for the momentum and energy balances.

This expression is general for any type of discontinuity in a flow, as long as the mean velocity of the discontinuity $\bar{u}_s$ is equal to zero. To go a step further in the analytical developments, it is necessary to make an hypothesis for the flame structure. In this study, a lean premixed flame is assumed (similar to [1]).

In this case, one expresses the heat release rate: $\dot{Q} = \rho_1\dot{V}\Phi h_s$, where ρ_1 is the density upstream of the flame, $\dot{V}$ the specific rate of volume consumption, Φ is the equivalence ratio and h_s the specific enthalpy of the premixture in stoichiometric conditions (J/kg). By differentiating and looking at the fluctuations, a kinematic equation is written. It links quantities upstream of the