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Circulation-controlled firewhirls with differential diffusion

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ABSTRACT

A flame-sheet theory for circulation-controlled firewhirls with differential diffusion is presented to investigate the effects of non-unity and unequal Lewis numbers on the flame shape and height of the firewhirls. Variable physical properties and a piecewise generalized power-law vortex model are implemented in the theory. For the fuel and oxidizer Lewis numbers being unequal but close to unity, the perturbation solutions of the Burke–Schumann-like transport equation for the Lewis-number-weighted coupling functions were obtained by using the Green's function method. The derived flame height expression not only confirms the previous discoveries, such as the Peclet number effect found by Chuah et al. (2011), the strong vortex effect by Klimenko and William (2013), and the variable density and mass diffusivity effects by Yu and Zhang (2017), but also demonstrates that the mass-diffusivity-ratio model correction newly proposed by Yu and Zhang (2017) is attributable to the leading-order non-unity Lewis number effect. The validity of the differential diffusion effects on the flame height was extended to arbitrary Lewis numbers and verified by means of the approximate far-field similarity solutions of the mixture fraction.

1. Introduction

Firewhirls are destructive natural phenomena that can be characterized as open diffusion flames intensified by strong vortices formed under suitable terrain and meteorological conditions [1–5]. Numerous theoretical and laboratory investigations have been carried out in the past for understanding the occurrence, mechanisms and behaviors of firewhirls [1–20]. Although buoyancy effects are considered generally indispensable to firewhirls, circulation-controlled firewhirls were found in nature [2,21,22] and reproduced in laboratory by Chuah et al. [3], who observed a sufficiently strong, inclined vortical flow generated a correspondingly oriented firewhirl over a liquid fuel containing pan, thus testifying the dominance of circulation over buoyance. To explain the experimental observation, they established a steady-state, axisymmetric, diffusion flame-sheet theory with the following major approximations:

1. the firewhirl has a large Peclet number so that it is significantly elongated along the axial direction where convection dominates over diffusion;
2. the vortical flow surrounding the firewhirl is modeled by a Burgers vortex, whose stream function is a quadratic function of the radial coordinate;
3. the flow has constant density and mass diffusivity;
4. the flow has a unity Lewis number.

By introducing the stream-function coordinates, Chuah et al. obtained a Burke–Schumann-like convection–diffusion equation for the mixture fraction, Z, as

$$\frac{\partial Z}{\partial \xi} = \frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta\frac{\partial Z}{\partial \eta}\right)$$

where the streamwise (the ξ-direction) convection is balanced by the traverse (the η-direction) diffusion. The flame height of the firewhirl is defined by the farthest axial location satisfying local stoichiometry, $Z = Z_{st}$. The flame height in physical coordinates, x_h, scaled by the diameter of the liquid fuel containing pan d_0, is given by

$$\frac{x_h}{d_0} = \frac{Pe}{16Z_{st}}$$

which verifies the experimental observation of $x_h/d_0 = O(Pe)$ because $16Z_{st}$ is near unity for common liquid fuels. Although the theory predicts the correct trend of the flame height increasing with Pe, it significantly underestimates the experimental results in the presence of strong vertical flows.

Postulating that the vortical flow surrounding the firewhirl cannot be properly modeled by the Burgers vortex, Klimenko and Williams established a theory by retaining the above approximations 1, 3 and 4 but replacing the approximation 2 with a strong vortex model [18,23]. The stream function of the strong vortex is

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a power function of the radial coordinate, and the power exponent α_v is smaller than two. The same Burke–Schumann-like transport equation for the mixture fraction was derived and a modified flame height expression [18] was given by

$$\frac{x_h}{d_0} = \frac{2}{\alpha_v}\frac{Pe}{16Z_{st}}$$

which explains the experimental data because the multiplicative factor, $2/\alpha_v > 1$, accounts for the additional stretching effect of the strong vortex in elongating the firewhirl.

The approximation 3 was adopted by both theoretical studies above, although its physical unreality was already recognized [18]. The temperature variation in the firewhirl flow field causes the corresponding variations of density and mass diffusivity, which may have significant influence on predictions of flame height. Yu and Zhang [19] established a theory with the approximations 1, 2 and 4 but discarded the approximation 3 by taking into account of temperature-dependent density and diffusivities. Unlike the earlier studies based on the mixture-fraction formulation, they adopted the coupling function formulation to obtain temperature solutions. A Howarth–Dorodnitsyn-like density-mass-diffusivity-weighted coordinate transformation was introduced to transform the coupling function formulation into the same Burke–Schumann-like transport equation. A revised flame height expression was accordingly given by

$$\frac{x_h}{d_0} = \left(\frac{T_m}{T_0}\right)^{2-\alpha_T}\frac{Pe}{16Z_{st}}$$

which also explains the experimental data [3]. Here the mean flame temperature, T_m, is higher than the liquid pool temperature, T_0, the exponent, α_T, characterizing the temperature dependence of mass diffusivity, is always smaller than 2. Consequently, the multiplicative factor, $(T_m/T_0)^{2-\alpha_T} > 1$, accounts for the physics that the temperature rise inside the firewhirl reduces the density and hence the inertia of the fuel vapor, which thereby can be advected to a higher altitude.

It has been recognized that the effects of variable physical properties and the strong vortex model are independent physical mechanisms for explaining the "enhanced" flame height [18,19]. As a result, integrating these two flame "enhancement" mechanisms in a theory must overshot the predictions of the flame height, and an additional flame "reduction" mechanism must exist to counteract the effects. Based on those considerations, Yu and Zhang [20] recently established a theory, in which the approximations 1 and 4 are retained, temperature-dependent physical properties and a generalized piece-wise power-law vortex model are adopted, and, in addition, the mass diffusivities on the fuel and oxidizer sides of the firewhirl are considered distinctly different (i.e. $D_F \neq D_O$). By use of approximate matching solutions to the species-enthalpy coupling functions with jumping mass diffusivities across the flame sheet, Yu and Zhang [20] derived an integrated expression of the flame height, consisting of four multiplicative factors, as

$$\frac{x_h}{d_0} = \alpha_D\frac{2}{\alpha_{v,\mathrm{eff}}}\left(\frac{T_m}{T_0}\right)^{2-\alpha_T}\frac{Pe}{16Z_{st}}$$

where the mass-diffusivity-ratio model correction, $\alpha_D = D_F/D_O < 1$, contributes a significant "reduction" mechanism for the flame height and therefore avoid the theoretical overshooting; $\alpha_{v,\mathrm{\ eff}} < 2$, an analog of α_v in Klimenko and William's theory, is an effective exponent for the piece-wise power-law vortex model.

In summary, of the four major approximations adopted in Chuah et al.'s theory, the experimentally verified large Peclet number approximation is indispensable for deriving the analytically solvable Burke–Schumann-like transport equation; the Burgers vortex model and constant physical properties have been examined and revised by Klimenko and Williams [18] and Yu and Zhang [19]; the unity-Lewis number assumption, which can seldom be exactly satisfied in combustion problems [24,25], has not be examined in all the previous theories. Therefore, the present study aims to theoretically investigate the effects of non-unity Lewis number on the circulation-controlled firewhirls. Furthermore, the previous study of Yu and Zhang [20] has revealed the "reduction" mechanism for the flame height owing to the mass-diffusivity-ratio model correction, which was however based on the approximate matching solutions. Consequently, the present study also attempts to give a mathematically rigorous treatment to further validate the model correction by addressing unequal Lewis numbers across the flame sheet. In this regard, the present study considers the effects of differential diffusion in general because the Lewis numbers are not only non-unity but also different across the flame sheet.

For a flame-sheet problem with non-unity and unequal Lewis numbers, the conventional coupling-function or mixture-fraction formulations are inapplicable. If the problem is one-dimensional, one can solve the convection–diffusion ODEs (ordinary differential equation) separately on the fuel and oxidizer sides and then match the solutions by using proper jumping conditions at the location of flame sheet, which is simultaneously determined by the matching. Although such a solution procedure has been successfully applied to droplet combustion [26,27] and other one-dimensional problems [24], it is inapplicable for two- or three-dimensional flame-sheet problems, in which the convection–diffusion PDEs (partial differential equation) cannot be analytically solved with the boundary conditions specified at the undetermined two- or three-dimensional flame sheet.

In the present paper, we shall first mathematically formulate a steady, axisymmetric circulation-controlled firewhirl system with non-unity and unequal Lewis numbers, in Section 2. We then present a perturbation theory to obtain mathematically rigorous solutions to the formulation by assuming the Lewis numbers on the fuel and oxidizer sides are close to unity, in Section 3. The flame shape and the flame height will be derived in explicit forms, and the effects of non-unity and unequal Lewis numbers will be discussed in detail for their physical implications, in Sections 4 and 5. Finally, we shall establish an approximate far-field similarity solution to a mixture-fraction formulation with arbitrary Lewis numbers, thus further verifying and extending the perturbation theory, in Section 6.

Nomenclature

Physical quantities

c_p	constant pressure specific heat
$d_0(r_0)$	diameter (radius) of the fuel liquid pool
D	mass diffusivity
q_c	heat of combustion per unit mass of fuel
q_v	latent heat of vaporization per unit mass of fuel
r, x, ϕ	cylindrical coordinates
T	temperature
u, v, w	velocity components in x, r, ϕ directions
W	molecular weight
Y	mass fraction
Z	mixture fraction
Z_{st}	stoichiometric mixture fraction
α_D	ratio of mass diffusivities, $\alpha_D = D_F/D_O$
α_{v1}	exponent in power-law vortex model (inside vortex core)
α_{v2}	exponent in power-law vortex model (outside vortex core)
α_v	overall exponent in power-law vortex model
α_T	parameter characterizing temperature-dependent mass diffusivity
α_Z	coefficient in the transport equation of mixture fraction
λ	thermal conductivity
ρ	density