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The effects of combined horizontal and vertical heterogeneity on the onset of convection in a porous medium with horizontal throughflow

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ABSTRACT

The effects of hydrodynamic and thermal heterogeneity, for the case of variation in both the horizontal and vertical directions, on the onset of convection in a horizontal layer of a saturated porous medium uniformly heated from below, with horizontal throughflow, are studied analytically for the case of weak heterogeneity. It is found that the horizontal throughflow has no effect on the stability of the longitudinal modes but it does affect the stability of the transverse modes. When the permeability decreases in the direction of the throughflow the transverse modes are stabilized (and so the longitudinal ones are favored). When the permeability increases in the direction of the throughflow a small amount of throughflow may destabilize the transverse modes and so destabilize the layer as a whole.

1. Introduction

The classical Horton–Rogers–Lapwood problem, for the onset of convection in a horizontal layer of a saturated porous medium uniformly heated from below, has been extensively studied. Early studies of the effects of heterogeneity in this situation are surveyed in Section 6.13 of Nield and Bejan [1]. The combined effects of vertical and horizontal heterogeneity of permeability, thermal conductivity and vertical temperature gradient have been treated recently by Nield and Kuznetsov [2–7] and Kuznetsov and Nield [8] for the case of weak heterogeneity, and this work has been summarized by Nield [9]. The cases of moderate and strong heterogeneity have been studied by Nield and Simmons [10], Nield and Kuznetsov [11], Nield et al. [12,13], Kuznetsov et al. [14] and Simmons et al. [15]. A transient problem has been discussed by Nield and Kuznetsov [16] and Kuznetsov et al. [17]. In this situation it is the heterogeneity of vertical temperature gradient that is involved. As far as the authors are aware the above papers are the only published analytical studies of the HRL problem with combined horizontal and vertical heterogeneity. A purely numerical simulation of a similar problem has been reported by Zhang et al. [18]. However, there is a large number of publications that address the effect of throughflow of thermal instability in the fluid (see, for example, [19–22]), which indicates a significant research interest to this area.

It appears that the effect of horizontal throughflow has hitherto not been studied. This situation involves an additional complexity in that the basic flow is now affected by heterogeneity of permeability.

2. Analysis

Single-phase flow in a saturated porous medium is considered. Asterisks are used to denote dimensional variables.

When discussing heterogeneity it is essential to have a clearly defined domain. Hence to be specific we consider a 3D rectangular box, $0 \leqslant x^* \leqslant L_x$, $0 \leqslant y^* \leqslant L_y$, $0 \leqslant z^* \leqslant H$, where the z^*-axis is in the upward vertical direction. We investigate a situation where the basic flow is unidirectional, in the positive x-direction.

The side walls are taken as insulated, and uniform temperatures T_0 and T_1 are imposed at the upper and lower boundaries, respectively. The problem is illustrated in Fig. 1.

Within this box the permeability is $K(x^*,y^*,z^*)$ and the overall (effective) thermal conductivity is $k(x^*,y^*,z^*)$. The Darcy velocity is denoted by $\mathbf{u}^* = (u^*, v^*, w^*)$. The Oberbeck–Boussinesq approximation is invoked. The equations representing the conservation of mass, Darcy's law, and the conservation of thermal energy, take the form

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{1}$$

$$u^* = -\frac{K(x^*,y^*,z^*)}{\mu}\frac{\partial P^*}{\partial x^*}, \tag{2a}$$

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Nomenclature

A	aspect ratio (height to width)
c	specific heat
H	layer depth
$\tilde{k}$	k/k_m
k	overall (effective) thermal conductivity
k_m	overall mean value of $k(x^*,y^*,z^*)$
$\tilde{K}$	K/K_m
K	permeability
K_{Hx}	harmonic mean value of $K(x^*,y^*,z^*)$ with respect to x^*
K_m	overall mean value of K
L_x	length of the domain in the x-direction
L_y	length of the domain in the y-direction
P	dimensionless pressure, $\frac{(\rho c)_f K_m}{\mu k_m} P^*$
P^*	pressure
P_0	pressure at $x^* = 0$
P_1	pressure at $x^* = L_x$
Ra	Rayleigh number, $\frac{(\rho c)_f \rho_0 g \beta K_m H (T_1 - T_0)}{\mu k_m}$
Ra_L	critical Rayleigh number for longitudinal modes
Ra_T	critical Rayleigh number for transverse modes
t^*	time
t	dimensionless time, $\frac{k_m}{(\rho c)_m H^2} t^*$
T^*	temperature
T_0	temperature at the upper boundary
T_1	temperature at the lower boundary
u	dimensionless horizontal velocity, $\frac{(\rho c)_m H}{k_m} u^*$
$\mathbf{u}^*$	vector of Darcy velocity, (u^*, v^*)
v	dimensionless vertical velocity, $\frac{(\rho c)_m H}{k_m} v^*$
V_0	dimensional mean velocity of the basic flow
x	dimensionless horizontal coordinate, x^*/H
x^*	horizontal coordinate
y	dimensionless upward vertical coordinate, y^*/H
y^*	upward vertical coordinate

Greek symbols

β	fluid volumetric expansion coefficient
γ_1, γ_2	parameters defined in Eq. (72)
θ	dimensionless temperature, $\frac{T^* - T_0}{T_1 - T_0}$
μ	fluid viscosity
ρ	density
ρ_0	fluid density at temperature T_0
σ	heat capacity ratio, $\frac{(\rho c)_m}{(\rho c)_f}$
ψ	streamfunction defined by Eq. (38)

Subscripts

B	basic solution
f	fluid
m	overall porous medium

Superscripts

$*$	dimensional variable

$$v^* = -\frac{K(x^*,y^*,z^*)}{\mu}\frac{\partial P^*}{\partial y^*}, \tag{2b}$$

$$w^* = \frac{K(x^*,y^*,z^*)}{\mu}\left[-\frac{\partial P^*}{\partial z^*} + \rho_0 \beta g (T^* - T_0)\right], \tag{2c}$$

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f \left[u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*}\right] = k(x^*,y^*,z^*)\left[\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right]. \tag{3}$$

Here $(\rho c)_m$ and $(\rho c)_f$ are the heat capacities of the overall porous medium and the fluid, respectively, μ is the fluid viscosity, ρ_0 is the fluid density at temperature T_0, and β is the volumetric expansion coefficient.

2.1. Unidirectional flow

Before we introduce dimensionless variables we will look at the consequences of requiring that the basic flow be unidirectional, so that $v^* = 0$ and $w^* = 0$. Now Eqs.(1) and (2a–2c) take the form

$$\frac{\partial u^*}{\partial x^*} = 0, \tag{4}$$

$$u^* = -\frac{K(x^*,y^*,z^*)}{\mu}\frac{\partial P^*}{\partial x^*}, \tag{5a}$$

$$\frac{\partial P^*}{\partial y^*} = 0, \tag{5b}$$

$$\frac{\partial P^*}{\partial z^*} = \rho_0 \beta g (T^* - T_0). \tag{5c}$$

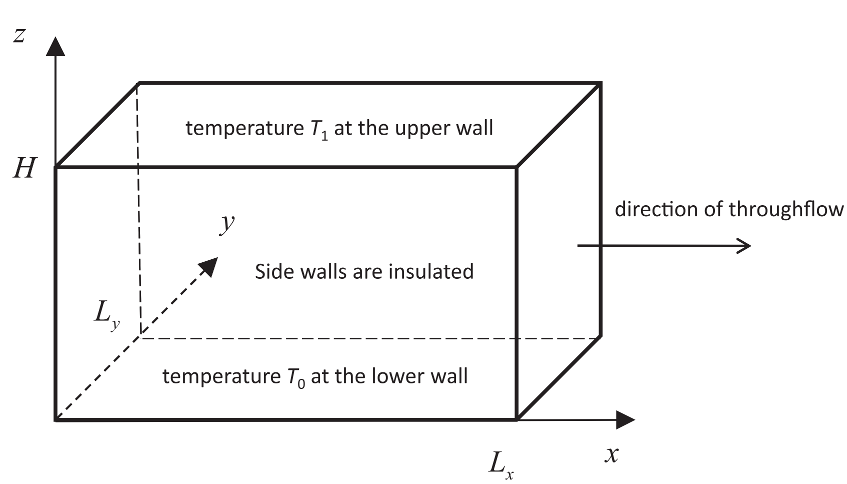

Fig. 1. Definition sketch.