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Structure and stability of premixed flames propagating in narrow channels of circular cross-section: Non-axisymmetric, pulsating and rotating flames

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Analysis of lean premixed freely propagating flames in narrow adiabatic channels of circular cross-section subject to a Poiseuille flow is carried out systematically within the context of a diffusive-thermal model. It is found that for flames with sufficiently low Lewis number multiplicity of solutions exists, in particular it is shown that for a given flow rate up to three different axisymmetric flames can coexist.

The rest of the study is focused on the influence of the Lewis number and the channel radius on the flame stability, linking first the results to the stability properties of corresponding planar flame fronts. The global stability analysis, giving accurate threshold values of instability, shows that for flames with Lewis number lower than one non-axisymmetric monotonous perturbations can grow, while for flames with Lewis number larger than one unstable axisymmetric and non-axisymmetric oscillatory modes can appear. An increase in the flow rate leads to a loss of stability of axisymmetric flames for $Le < 1$ while for flames with $Le > 1$ the Poiseuille flow produces a stabilization effect. This analysis seems to have received no attention in the literature for flames in circular channels.

Finally, the stability results are compared successfully with three-dimensional unsteady numerical simulations. They show that for $Le < 1$ non-axisymmetric flames propagating with a constant velocity appear for positive values of the flow rate. For flames with $Le > 1$, depending on the channel width, pulsating axisymmetric and rotating non-axisymmetric flames arise. It is demonstrated that when the non-axisymmetric oscillatory perturbations are the most unstable, the flame adopts a multi-headed non-axisymmetric shape and can propagate in a rotating manner while the central part of the flame advances at a constant velocity.

1. Introduction

The propagation of premixed flames in channels fills a highly important place in combustion studies. Apart from traditional interest related mainly to safety, that is, to the determination of whether or not a flame can ignite and propagate along a conduct filled with a fuel and oxidizer mixture, flame propagation in narrow channels has received renewed attention in the literature, due to the role it plays in new technologies using micro combustion devices [1–3].

The flame propagation in channels comprises different physical effects which act simultaneously, hampering comprehension of the phenomenon as a whole. The thermal expansion producing the fluid–flame interaction, the flame–wall heat exchange, the differential diffusion effect or chemical complexities are among the most important ones. A possible way to get a more clear physical insight into the problem is to try to understand the role of every effect separately by simplifying the formulation. Perhaps some simplifications may create difficulties when comparing the model and the experimental results. Nevertheless, the gain in physical understanding obtained from simplified models is significant, which is confirmed by the existence of an extensive hierarchy of models employed in the combustion science.

Following this line, several studies have been undertaken in recent years on the effect of the flow rate and channel width [4] or differential diffusion [5–9] in the propagation of flames in narrow channels, using a diffusive-thermal model. Other studies in the same line analyzed the effect of heat losses to the wall [10,11]. Apart from pioneering numerical analysis in the past [12,13], the effect of thermal expansion was taken into account in the framework of the complete Navier–Stokes equations in [14–20,22–24]. It should be noted that in [15–17,20] flames propagating in

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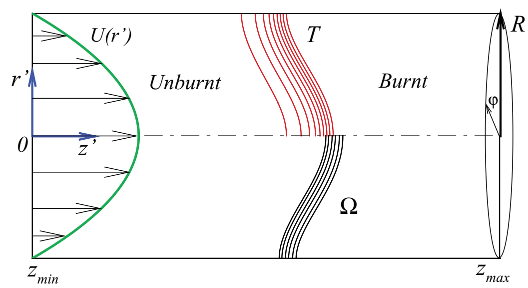

Fig. 1. Schematic of the channel configuration, parabolic velocity field (with $U_0 > 0$), typical isotherms (upper half, at intervals 0.1) and reaction rate contours (lower half) calculated for $Le = 1$, $m = 0.5$ and $d = 40$.

semi-infinite channels closed at their ignition end were investigated and the wall friction effect was identified as the driving mechanism for the flame acceleration.

The studies on differential diffusion effects have in particular shown a rich flame dynamics even in planar channels [8,11]: when the Lewis number is different from one, diffusive-thermal instabilities appear. When $Le < 1$ instability to non-symmetric perturbations results in the steady stable flame being non-symmetric. For $Le > 1$, oscillatory symmetric and non-symmetric solutions can appear. This has important implications in the flame propagation properties and flashback critical conditions.

However, in a significant number of the numerical studies dedicated to freely propagating flames in narrow channels of circular cross-section, because of computational cost reasons, it was assumed from the start that the flame shape should be symmetric with respect to the axis [4,6,10,12–17,19]. This restriction in symmetry for freely propagating flames was removed only in [5,18,21], where it was shown that the effects of symmetry breaking for flames with Le different than one were also present in cylindrical ducts.

In the present paper we consider the case of a prescribed reactants flow rate through an (infinite) channel, extending our aforementioned works [8,11]. We study systematically the free propagation of flames in narrow adiabatic channels of circular cross-section within the context of a diffusive-thermal model with the objective of analyzing the effects of differential diffusion, the channel radius and the flow rate on the shape, propagation speed and stability of the flame. To this end, in Section 2 we give the general formulation; the numerical treatment is briefly described in Section 3; the numerical results describing the axisymmetric flames are presented in Section 4; the global linear stability analysis formulation is given in Section 5; the stability of flames propagating through a stagnant mixture is described in Section 6; the global stability results presented in Section 7, with the results of direct three-dimensional calculations in Section 8. Finally, concluding remarks are drawn in the last section.

2. General formulation

A combustible mixture at initial temperature T_0 flows in an infinitely long channel of circular cross-section with mean velocity U_0. In what follows we use the standard cylindrical coordinates with z', r' and φ denoting the longitudinal, radial and angular coordinates, respectively, and t' denoting the time. Primes here and hereafter indicate dimensional quantities if the same notation is used for dimensional and non-dimensional variables. A sketch of the geometry and the coordinate system is given in Fig. 1. In all cases considered below the fresh cold mixture is situated at the left of the flame (at $z' \to -\infty$) while the flow may be either directed to the right with $U_0 > 0$, as shown in Fig. 1, or to the left with $U_0 < 0$. The flame can propagate, depending on the parameters, in both directions, to the right ($z' \to \infty$) or to the left ($z' \to -\infty$) relative to the wall.

This work deals with a diffusive-thermal or "constant-density" model, according to which the density of the mixture ρ, the heat capacity c_p, the thermal diffusivity $\mathcal{D}_T$, and the molecular diffusivity $\mathcal{D}$ are all assumed constant. Consequently, the flow field, unaffected by the combustion process, is given by the Poiseuille flow, $u_z = 2U_0[1 - (r'/R)^2]$, $u_r = u_\varphi = 0$, where R is the radius of the channel. The present study is carried out for cases of circular channels with adiabatic walls, in order to remove the effects of the flame–wall heat exchange, which will be reported elsewhere. The heat-loss effects were reported recently for planar channels in [11]. In practical conditions good insulation may be attained in a double walled vacuum duct [25].

In the present study we use a simplified kinetic model where the combustible mixture undergoes a chemical reaction modeled by a global irreversible step $F + O \to P + Q$, where F, O and P denote the fuel, the oxidizer and the products, respectively, and Q is the heat released per unit mass of fuel. The amount of fuel consumed in moles per unit volume, per unit time, is given by $\Omega \sim (\frac{\rho Y_F}{W_F})(\frac{\rho Y_O}{W_O})\exp(-E/\mathcal{R}T)$, where Y_F, Y_O are the mass fractions and W_F, W_O are the molecular weights of the fuel and oxidizer, respectively, ρ is the density of the mixture, E is the overall activation energy and $\mathcal{R}$ is the universal gas constant. Assuming that the mixture is lean in fuel, the oxidizer mass fraction remains nearly constant and $\Omega = \mathcal{B}\rho^2 Y_F \exp(-E/\mathcal{R}T)$, where $\mathcal{B}$ is a pre-exponential factor containing Y_O and the molecular weights.

In order to describe the flame propagation a reference frame attached to the flame is used, much as it was done in [5,8,11]. Following the temperature distribution along the axis of the channel, starting from the unburned side, we choose the *first* point $z' = z'_*$ where the temperature is equal to some value $T = T_*$ (the reference temperature below). In the following, the reference frame is attached to this point. The velocity $U_f(t')$ of this point relative to the wall as a function of time characterizes the time-dependent development of the combustion process. In the case of steady flame propagation the whole flame propagates as an invariable structure with a constant velocity equal to U_f.

The burning velocity of the planar flame, S_L, and the thermal flame thickness defined as $\delta_T = \mathcal{D}_T/S_L$ are used below to specify the non-dimensional parameters. The non-dimensional temperature is defined as $\theta = (T - T_0)/(T_e - T_0)$, where $T_e = T_0 + QY_{F0}/c_p$ is the adiabatic temperature of the planar flame based on the unburned gas temperature T_0 and the upstream fuel mass fraction Y_{F0}. Choosing R and $R^2/\mathcal{D}_T$ as the reference length and time scales, $(z, r) = (z'/R, r'/R)$, $t = t'/(R^2/\mathcal{D}_T)$, and Y_{F0} to normalize the fuel mass fraction, $Y = Y_F/Y_{F0}$, the dimensionless equations written in the moving reference frame become

$$\frac{\partial\theta}{\partial t} + \sqrt{d}[u_f(t) + 2m(1 - r^2)]\frac{\partial\theta}{\partial z} = \frac{\partial^2\theta}{\partial z^2} + \Delta_{r\varphi}\theta + d\,\omega(Y, \theta), \tag{1}$$

$$\frac{\partial Y}{\partial t} + \sqrt{d}[u_f(t) + 2m(1 - r^2)]\frac{\partial Y}{\partial z} = \frac{1}{Le}\left(\frac{\partial^2 Y}{\partial z^2} + \Delta_{r\varphi}Y\right) - d\,\omega(Y, \theta), \tag{2}$$

where $\Delta_{r\varphi} = \partial^2/\partial r^2 + r^{-1}\partial/\partial r + r^{-2}\partial^2/\partial\varphi^2$ and

$$\omega(Y, \theta) = \frac{\beta^2}{2Le u_p^2} Y \exp\left\{\frac{\beta(\theta - 1)}{1 + \gamma(\theta - 1)}\right\} \tag{3}$$

is the non-dimensional reaction rate.

The following non-dimensional parameters appear in Eqs. (1)–(3): the Zel'dovich number, $\beta = E(T_e - T_0)/\mathcal{R}T_e^2$, the Lewis number, $Le = \mathcal{D}_T/\mathcal{D}$, the heat release parameter, $\gamma = (T_e - T_0)/T_e$, the reduced Damköhler number, $d = R^2 S_L^2/\mathcal{D}_T^2 = R^2/\delta_T^2$ and the