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# Effect of axial diffusion on the response of diffusion flames to axial flow perturbations☆

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ABSTRACT

This paper elucidates the behavior and dynamics of non-premixed flames responding to bulk fluctuations in flow velocity. It expands previous work on this problem by consistently incorporating finite Peclet number ($Pe = U_0 R_f / \mathscr{D}$) effects, and differentiating inflow boundary and dynamical effects on the flame dynamics. For analytical tractability, prior treatments of this problem generally prescribe the inflow boundary conditions into the domain. This paper shows, however, that prescribed inflow conditions, such as a step or constant local diffusive flux boundary condition, neglect axial diffusion effects in the region where their effects are most important; i.e., in the near-burner exit region where high transverse gradients and mass burning rates control the heat release dynamics. As the burning rate of non-premixed flames are controlled by mixture fraction gradients, the influence of axial diffusion substantively influences several burning rate characteristics of the flame. In addition, these effects cause the leading edge position of the flame front to oscillate, even for infinitely fast chemistry. Even in $Pe \gg 1$ flames, axial diffusion introduces several fundamentally new features to the problem, resulting in exponentially decaying, dispersive flame wrinkle propagation with downstream distance. Also investigated here are asymptotic results for the heat release dynamics. It is shown that axial diffusion introduces a triple-zone asymptotic structure into the unsteady heat release characteristics, resulting in O(1) flame transfer function trends for $St \ll 1$ (to which an $n$–$\tau$ model is developed), O($1/St^{1/2}$) for intermediate Strouhal numbers, and O($1/St$) for high Strouhal numbers. Finally, it is shown that the phase of the heat release response is approximately half that of a premixed flame with the same length, due to the concentration of unsteady heat release near the burner outlet, where transverse gradients are largest.



## 1. Introduction

This paper investigates the features and dynamics of non-premixed flames responding to spatially uniform fluctuations in flow velocity. It is motivated by performance and safety concerns arising from combustion instabilities, which manifest as large amplitude heat release oscillations associated with one or more combustor acoustic modes [1]. This study concentrates on the velocity-coupled part of the feedback loop, analyzing the flames response to flow disturbances. The response of premixed flames to flow disturbances and the combustion instability characteristics of lean, premixed combustors has been extensively investigated and documented over the last few decades [1–8]. In contrast, the behavior of non-premixed flames responding to flow disturbances, both in regards to the space-time reaction sheet and temporally varying heat release dynamics, both local and spatially integrated, has received significantly less attention. Non-premixed, or partially premixed flames, however, are routinely encountered in liquid-fueled combustion systems, as well as in pilot systems for premixed combustors, and industrial operators routinely note the profound influence of non-premixed pilot fuel on combustor oscillation limits.

A number of studies have delved into the natural dynamics of non-premixed flames. In the buoyancy dominated regime where the Froude number, $Fr = U_0^2/(gR_f)$, is not too high, these flames are globally unstable and exhibit narrowband oscillations due to the periodic generation and traveling of vortical structures vertically along the flame. These are manifested as flame flicker at a low frequency (∼12 Hz) that is remarkably insensitive to flow rate, burner size, and gas composition [9,10]. The amplitude of spatial flickering is, however, a function of these parameters. Recent studies have shown that this global instability disappears at small Froude numbers, or when the flame becomes momentum dominated at large Froude numbers [11].

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**Nomenclature**

| | |
|---|---|
| $\mathscr{D}$ | diffusion coefficient |
| $L_f$ | flame length |
| $Pe$ | Peclet number, $Pe = U_0 R_f/\mathscr{D}$ |
| $\dot{Q}$ | spatially integrated heat release |
| $R_w$, $R_f$ | radial distance to confining wall and fuel port, respectively |
| $St_{L_f}$,$St_R$ | Strouhal number based on flame length, $St_{L_f} = f L_f/U_0$, and half-burner width, $St_R = f R_f/U_0$ |
| $U_0$ | steady axial flow velocity |
| $Y$ | species mass fraction |
| $\tilde{Z}$ | true mixture fraction |
| $Z$ | rescaled mixture fraction |
| $f$ | forcing frequency |
| g | gravitational force |
| $s_d$, $s_c$ | flame displacement and consumption speed, respectively |
| $u$ | axial flow velocity |
| $x$, $y$ | axial and radial coordinate, respectively |
| $\varepsilon$ | small perturbation parameter |
| $\theta$ | local angle of flame surface with respect to axial coordinate |
| $\xi$ | flame position |
| $\varphi_{ox}$ | stoichiometric mass ratio of oxidizer to fuel |
| $\omega$ | angular frequency |
| $(\hat{\ })$ | frequency domain variable |
| $()_{st}$ | stoichiometric value |
| $()_0$ | mean/steady state component |
| $()_1$ | fluctuating component |

External excitation of non-premixed flames, such as by acoustic forcing, has also been studied extensively, often with the motivation of enhancing mixing and/or decreasing pollutant emissions. When subjected to external excitation, lower Froude number, nominally unstable flames exhibit a variety of response features that depend upon the frequency and magnitude of the excitation. For example, Chen et al. studied the response of a non-premixed flame exposed to acoustic excitation [12], showing oscillations in both the fuel jet flow and flame sheet position, both of which were dependent upon the forcing frequency and amplitude. They and others [13–15] also showed nonlinear behavior, such as the presence of sum and difference frequencies of the buoyant instability and external forcing frequencies, subharmonics and harmonics of the excitation frequency, and frequency locking – i.e., the disappearance of oscillations associated with the natural buoyant instability at sufficient excitation amplitudes. For example, Williams et al. [16] explored this lock-in behavior, showing that forcing the fuel stream at a frequency close to the natural buoyant instability frequency was accompanied by the presence of large vortices on the air side of the flame, coupling the overall flame response to the forcing frequency. They also observed that a related lock-in phenomenon could happen at the first subharmonic of the forcing frequency, when the forcing frequency was close to twice the natural instability frequency.

As a result of the strong effect of forcing on the ambient/coflowing air and its entrainment with the fuel jet, a number of studies have also noted significant influences on soot and $NO_x$ production from the flame [17–19] – sensitivities which are much stronger in non-premixed flames than in premixed flames. For example, Saito et al. [17] showed that soot can be suppressed in acoustically excited non-premixed flames, with reductions of up to 50% in a laminar flame, and 90% for a turbulent flame.

Additional studies have looked into the dynamics of laminar, momentum dominated flames, focusing on the flame's space-time dynamics due to velocity and equivalence ratio perturbations. Forced flow disturbances produce a spatially varying, oscillatory velocity field whose component that is normal to the flame causes wrinkling and pocket formation beyond critical conditions, as well as oscillatory reaction and heat release rates [19–21].

A number of analytical studies have also considered the response of momentum dominated non-premixed flames. A significant theoretical literature on the unforced problem exists, the Burke-Schumann flame being a classic problem, along with works distinguishing the effects of streamwise diffusion [22, 23]. Several treatments of the forced, unsteady problem have been reported, in particular those of Sujith [24–26], Chakravarthy [21, 26], Juniper et al. [27], and Magina et al. [28,29]. These studies have analyzed this problem within the infinite reaction rate, $Z$-equation formulation for the mixture fraction. Solutions were developed for the flame position and heat release for several problems, including the flame response to axial velocity and mixture fraction oscillations.

This study closely follows the work of Magina et al. [28,30,31] which primarily focused on large Peclet number flames, i.e. where axial transport is dominated by convection relative to diffusion, a limit which significantly simplifies the analysis and enables the development of explicit solutions for the space-time dynamics of the flame position and unsteady heat release. It was shown that the oscillatory flow velocity component normal to the flame was responsible for exciting wrinkles on the flame sheet, whose magnitude had strong axial dependence and monotonically decreasing phase. In the absence of axial diffusion, these wrinkles convect axially at the mean flow speed, $U_0$, with constant amplitude and phase speed. Axial diffusion effects cause these wrinkles to dissipate as they convect downstream, reducing the overall wrinkle amplitude and resulting in imperfect interference. Finally, they showed that the spatially integrated unsteady heat release was dominated by mass burning rate fluctuations, rather than area fluctuations, and rolled off much slower with frequency, $O(St^{-1/2})$ compared to $O(St^{-1})$ for premixed flames, indicating non-premixed flames are much more sensitive to flow perturbations at intermediate Strouhal numbers.

Several questions still remain about the flame position and heat release dynamics of finite Peclet number flames. In particular, axial diffusion effects manifest themselves in a variety of ways, not all of which have been captured in prior analyses. Most theoretical analyses of the problem impose inflow conditions on the mean and fluctuating solutions, even in studies that capture axial diffusion effects within the domain itself. For example, our earlier study that demonstrated how axial diffusion introduced damping of flame wrinkles utilized a step-inlet boundary condition [29]. This simplification introduces a singularity in the solution, as there is an infinite gradient in mixture fraction at the fuel port lip. As we will show here, the high frequency characteristics of the heat release are quite sensitive to the inflow profile, and the step-inflow boundary condition leads to incorrect conclusions on these asymptotic characteristics of the heat release transfer function, even in the $Pe \gg 1$ limit. Stated differently, specifying an inflow step boundary conditions neglects axial diffusion effects in the region where these effects are most important – in the near-burner exit region where high transverse gradients and mass burning rates control the heat release dynamics. Thus, a key contribution of the present study is to consistently capture finite $Pe$ effects. As will be shown, this requires computational solutions of the governing equations, as explicit analytical solutions are not possible in this case.

A recent paper by Xiong et al. [32] on buoyancy dominated flames has also emphasized the significance of the inflow conditions on coflow diffusion flame solutions. Similar to this study, they specified the fuel inlet boundary condition well upstream of