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# Nonlinear analysis of an acoustically excited laminar premixed flame

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ABSTRACT

The nonlinear effects of the fully coupled two-way interactions between a disturbance field defined by an amplitude variation of the inflow velocity distribution and a laminar premixed flame incorporating gas expansion effects are investigated by numerically solving the conservation equations of a compressible fluid. Results of the higher harmonics of the flame front perturbation for two burnt to unburnt temperature ratios show how the nonlinear transfer of flame response is affected by the hydrodynamic instability and the shear layer effect. The flame response at the fundamental forcing frequency, i.e., the flame-describing function can be well predicted by the reduced-order model (ROM) for both investigated temperature ratios at low Markstein length values, where the balance between the flame stretch effects and the hydrodynamic instability is insignificant. However, at high Markstein lengths the balance between the flame stretch and the hydrodynamic instability influences the flame response characteristics at all excitation amplitudes. The nonlinear study of the flame front perturbation indicates that differences in the higher harmonic content of the overall nonlinear flame response between the detailed computation and the ROM predictions are located at the flame front just before the flame pockets separate. Thus, the shear layer and the hydrodynamic instability effect determine the nonlinear character of the flame response through the nonlinearity introduced at the flame base, the location of flame pocket separation, and the annihilation process at the flame tip especially at high Markstein length values. In addition, the flame base movement response is compared to a recent developed reduced-order flame base model to understand the findings with respect to the assumption of flame attachment at the burner lips.

## 1. Introduction

Combustion instabilities can cause serious problems which limit the operating envelope of low-emission lean premixed combustion systems. These instabilities frequently occur due to the coupling of the acoustic modes with the unsteady heat release of the flames. This phenomenon can result in high amplitude pressure oscillations that cause, e.g., excessive pollutant emissions or even hardware damage [1]. Thus, a key requirement to predict the onset and evolution of these instabilities is a model for the response of a premixed flame to velocity fluctuations generated by acoustic waves. There have been significant efforts over the past decades from various groups which have developed analytical solutions for the transfer function of the heat release rate of the flame and the imposed acoustic velocity fluctuations [2–9]. These analytical solutions for the flame-transfer function, however, neglected the influence of gas expansion at the flame surface on the flame motion. This assumption, however, implies that the Darrieus–Landau (DL) instability, which is caused by the preferential acceleration of the flow normal to the flame surface in regions that are concavely curved relative to upstream reactants and vice versa [10,11], has no effect on the flame-transfer function. Since the DL instability influences the flame response at the fundamental forcing frequency for a low excitation amplitude, as discussed in [12], this DL instability might influence also the nonlinearity of the system inducing a considerable flame response at higher harmonic frequencies for large excitation amplitudes. A detailed discussion of the literature with respect to the linear concept of flame response is given in [12].

In the following, the nonlinear flame response and its relation to the DL instability mechanism and shear layer is briefly discussed. The DL instability leads to an increased amplitude of the flame surface perturbation [10,11]. Thus, a large change in the characteristics of the heat release transfer function is expected when the associated flame surface perturbation changes from being stable to unstable.

In addition to the convective DL instability, the flame wrinkling evolution is influenced by the disturbance field which introduces a change in the vorticity distribution depending on the gas expansion

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ratio [12], the streamline curvature, and the varying flame front position [13–18]. Thus, the fully coupled two-way interaction between the disturbance field influenced by the vortices generated in the shear layer near the flame base and the flame determines how the flame wrinkles develop when traveling from the flame base to the tip [15–17,19]. Although many studies focus on the linear concept of flame response, the flame responds additionally at the higher harmonic frequencies which become important at increasing excitation amplitude [8,20].

In numerous investigations the thermoacoustic response of laminar premixed flames in a duct [4,6,21–23] is addressed. Kashinath et al. [21] mentioned that the higher harmonic content of the nonlinear dynamics can lead to a limit cycle of large amplitude for a ducted flame. Lieuwen et al. [8] numerically analyzed a conical and a wedge flame and showed that the pocket formation and separation plays an important role in the flame response saturation for wedge flames. For the conical flame set-up they showed an almost linear flame response, however, the theoretical analysis assumed zero gas expansion effects, a constant burning velocity and a uniform velocity field. Thus, the nonlinearities caused by the coupled two-way interaction between the flame and the flow are neglected, although the two-way exchange changes the flame kinematic response with increasing excitation amplitude and frequency due to the augmenting curvature effects [24,25] and hence amplifies the Darrieus–Landau instability [10,25]. Balachandran et al. [20] studied a turbulent bluff-body stabilized premixed flame under imposed velocity perturbations. They found that for inlet velocity amplitudes greater than 15% the heat release response increases nonlinearly induced by the shear layer which rolled up into vortices. These vortices generated cusp-like formations and annihilation events inducing energy transfer to the higher harmonics of the forcing frequency. Blanchard et al. [18] studied in a detailed experimental and numerical analysis the flame response of an M-flame, where for low frequency excitation the convective mode dominates ($St < 6.3$), for mid-range Strouhal numbers ($6.3 < St < 32$) the convective and acoustic modes coexist, and for higher Strouhal numbers ($St > 32$) a pure acoustic mode is dominant. Durox et al. [26] analyzed the conical flame with the flame-describing function and showed that the flame response saturation is increased at growing excitation amplitude and frequency due to the flame stretch effects inducing separated pockets. However, the transfer to the higher harmonic content which is caused by this nonlinearity was not investigated in detail. In addition, Karimi et al. [27] investigated the flame response at high amplitude forcing and observed a lower location of maximum flame front amplitude when the forcing level exceeds the linear regime > 15%. They also mentioned that the flame base movement becomes significant depending on the frequency and excitation amplitude. Cuquel et al. [28] analyzed this flame base movement response and developed a model to capture its contribution to the flame response. This reduced-order model captures the phase of the flame foot oscillation quite well, however, the amplitude was overpredicted. In a recent study by Mejia et al. [29], the influence of the wall temperature influence on this flame base kinematics is analyzed. They found that the flame response is globally influenced by the temperature at the wall through the changed flame base movement response. Thus, a fundamental understanding of the nonlinearity with respect to the vortices generated in the shear layer near the flame base, which is denoted shear layer effects, and the balance between the hydrodynamic instability and flame stretch is still missing. The impact of this nonlinearity should be investigated to fully understand the nonlinear flame response with increasing excitation amplitude.

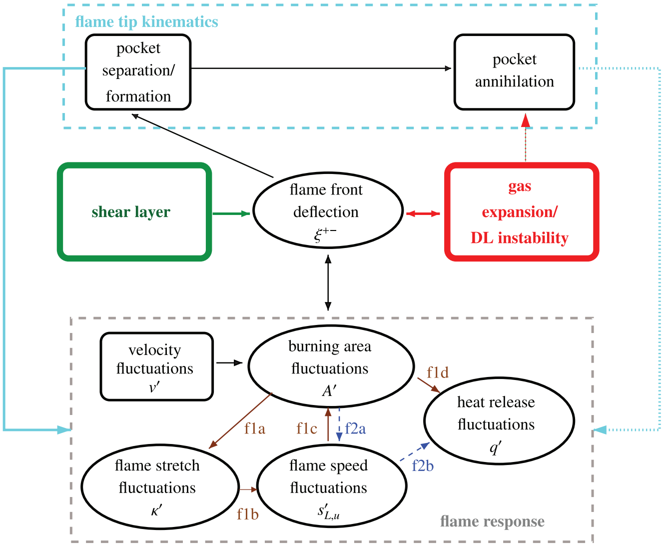

**Fig. 1.** Direct influence of gas expansion (red box) and shear layer effects (green box) and indirect influence via the flame tip kinematics (light blue dashed box) on the flame response (gray dashed box) [12]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

Considering the results discussed in [12], Fig. 1 summarizes the influence of the shear layer effect (green box) and the gas expansion effect (red box) on the flame response (gray dashed box) for a moderate excitation amplitude $\epsilon = 0.1$. The flame response differences of a reduced-order model (ROM), which neglects the gas expansion and the shear layer, to a detailed solution based on the full conservation equations can be explained with respect to the missing red and green box. The ROM includes neither gas expansion (red box) nor shear layer effects (green box) such that the flame response discrepancies are evident due to these two additional boxes. Based on the findings from [12] the following list summarizes the influences on the flame response (gray dashed box) that are not captured by the ROM.

(i) Gas expansion (red box):
  - It induces the Darrieus–Landau mechanism resulting in an increasing flame front deflection $\xi^{+-}$ which changes the flame tip kinematics (light blue dashed box) of the detailed simulation compared to the ROM data. A change in the flame tip kinematics causes the burning area fluctuations $A'$ to increase/decrease depending on the flame pocket separation location and the flame pocket size (light blue solid line).
  - It influences the flow field expansion causing the flame pocket annihilation speed to increase (red dotted line) which results in differences to the ROM with respect to the flame response due to increased flame stretch oscillations $\kappa'$ and with respect to the phase response due to the temporally varying pocket annihilation (light blue dashed line).

(ii) Shear layer (green box):
  - It induces an asymmetric initial flame front deflection $\xi^{+-}$ in the detailed solution which changes the burning area $A'$ and the flame stretch oscillations $\kappa'$.
  - It contributes via convection to the flame response and the effect on the flame response increases at decreasing gas expansion ratio.

In this study, the conservation equations of a compressible fluid are solved for a two-dimensional slot-stabilized premixed flame for different burnt to unburnt gas temperature ratios of $2 \le T_b/T_u \le 7$ and several excitation amplitudes $0.05 \le \epsilon \le 0.5$ to analyze the effect of gas expansion and excitation amplitudes on the nonlinear flame response characteristics. These full conservation equation (FCE) results are compared with the corresponding flame front and flame heat release response characteristics determined by reduced-order model (ROM) computations using LSGEN2D introduced in [30,31]. In addition, the flame base movement response is compared to a reduced-order flame base model (ROFM) developed in a recent study

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