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Heat transfer to a sphere in tube flow of power-law liquids

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ABSTRACT

Extensive new results on forced convection heat transfer from an isothermal heated sphere exposed to the fully developed laminar velocity profile of power-law fluids in a tube are reported herein. In particular, these results endeavor to elucidate the influence of the pertinent dimensionless governing parameters, namely, Reynolds number ($5 \leqslant Re \leqslant 100$), Prandtl number ($1 \leqslant Pr \leqslant 100$), power-law index ($0.2 \leqslant n \leqslant 1$) and sphere-to-tube diameter ratio ($0.05 \leqslant \lambda \leqslant 0.5$). The heat transfer characteristics are analyzed in terms of the local as well as surface averaged values of the Nusselt number. Broadly, all else being equal, shear-thinning behavior promotes heat transfer, though confinement limits such an increase in shear-thinning fluids depending upon the value of the power-law index. Similarly, all else being equal, confining walls also enhance the rate of heat transfer which is, however, maximum in the case of Newtonian fluids. Possible reasons for such counter-intuitive trends are advanced. The present numerical results have been approximated by simple expressions thereby enabling their interpolation for the intermediate values of the governing parameters like Reynolds number, Prandtl number, diameter ratio and power-law index.

1. Introduction

Heat transfer from a spherical particle suspended in moving fluid streams represents an idealization of several industrially important processes. Typical examples include fluidized and fixed bed reactors, slurry and bubble column reactors employed to carry out a range of industrially important chemical reactions and other contacting operations. Further examples abound in the processing and handling of solid–liquid suspensions in food [1], coal liquefaction, suspension polymerization, catalytic hydrogenation, pharmaceutical and personal care product manufacturing sectors. For instance, food particles are given thermal treatment via the flow of their suspensions in carboxymethyl cellulose and xanthan gum solutions (both of which exhibit shear-thinning behavior) in heated tubes, e.g., see [2]. Similarly, polymeric additives are used as thickening agents in the formulation of lotions, gels and creams which should meet the prescribed stability criteria against temperature without compromising their functionality [3,4]. Admittedly, while most of the aforementioned and other potential applications entail swarms of particles which may or may not be spherical in shape, it is readily conceded that a satisfactory understanding of the behavior of a single sphere often serves as a launching pad to undertake the modeling of real life applications. Even for a single sphere, the rate of heat transfer between the particle and the ambient fluid is influenced by a large number of variables like the physical properties of the fluid medium and their temperature-dependence, diameter and positioning of the particle, flow rate of the fluid and whether the flow is confined or unconfined. Further complications arise from the type of the regime (free or mixed or forced convection) as well as the type of the thermal boundary condition (uniform heat flux or temperature) prescribed on the surface of the sphere and that on the tube wall. Thus, there are simply too many variables exerting varying levels of influence on the prevailing temperature field and the resulting rate of heat transfer between the sphere and the ambient fluid. Consequently, over the years, a significant body of knowledge has accrued on various aspects of heat transfer between a sphere and an ambient fluid, especially for the case of simple Newtonian fluids. Excellent state of the art reviews summarizing the pertinent literature are available [5–9]. Suffice it to say here that it is now possible to estimate the value of the convective heat transfer coefficient for a sphere placed in an unconfined fluid medium over most ranges of conditions of practical interest. However, in many applications, the flow occurs in finite size pipes and vessels and the flow is thus confined. While if the diameter of the pipe or vessel is much larger than that of the sphere, confinement exerts very little influence, at least as far as the gross parameters like drag coefficient and Nusselt number, etc. are concerned. On the other hand, strictly speaking, the confinement, how so ever small, modifies the detailed velocity and temperature fields due to the no-slip boundary condition implemented on the solid wall. Naturally, this effect gets

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Nomenclature

C_p	heat capacity, J/kg K
d	sphere diameter, m
D	tube diameter, m
g	acceleration due to gravity, m/s^2
Gr	Grashof number, dimensionless
I_2	second invariant of the rate of deformation tensor, dimensionless
h	heat transfer coefficient, W/m^2 K
k	thermal conductivity of fluid, W/m K
L_d	downstream distance, dimensionless
L_u	upstream distance, dimensionless
m	power-law consistency index, Pa s^n
n	power-law index, dimensionless
n_s	unit normal vector on the surface of the sphere
Nu	Nusselt number, dimensionless
p	isotropic pressure, Pa
Pe	Peclet number, dimensionless
Pr	Prandtl number, dimensionless
R	radius of sphere, m
R_o	radius of tube, m
Re	Reynolds number, dimensionless
r, z	coordinates, m
T	fluid temperature, K
T_o	fluid temperature at the inlet, K
T_w	constant temperature at the surface of sphere, K
T^*	dimensionless temperature
$\mathbf{V}$	velocity vector, m/s
V_{max}	maximum velocity at the center of the tube, m/s
$\langle V \rangle$	area average velocity, m/s

Greek symbols

α	thermal diffusivity, m^2/s
ε	rate of deformation tensor, s^{-1}
η	viscosity, Pa s
λ	diameter ratio (=d/D), dimensionless
θ	angular position on the surface of the sphere measured from the front stagnation point, dimensionless
ρ	fluid density, kg/m^3
τ	extra stress tensor, Pa

Superscripts and subscripts

b	bulk
T	transpose
w	sphere surface

accentuated with the increasing sphere-to-tube diameter ratio. Therefore, in general, the influence of the confining wall can not be neglected because of the modifications to the prevailing velocity and temperature profiles across the tube diameter. For instance, thermal processing of foodstuffs is often carried out in horizontal, vertical or inclined tubes in which the particle-to-tube diameter ratio may be of the order of ~0.3–0.4 [2]. Similarly, it is not uncommon to encounter particle-to-tube diameter ratios as high as 0.3–0.35 in hydraulic transport of coarse particles in non-Newtonian carrier fluids which are subject to variable external temperature conditions which, in turn, can alter rheological properties of the carrier thereby influencing the value of pressure gradient across the pipe length [10]. In the present case, the prevailing velocity distribution across the cross-section of the tube further influences the velocity and temperature gradients in the vicinity of the sphere which, in turn, directly influences the rate of momentum [11] and heat transfer between the fluid and the submerged sphere. Intuitively, owing to the application of the no-slip condition on the tube wall, one would expect the velocity and temperature gradients on the surface of the sphere to become steeper thereby leading to augmentation in both drag and heat transfer with reference to that for an unconfined sphere. Such an enhancement has been reported for cylinders in air and water by Perkins and Leppert [12,13] and for model food particles by Sastry et al. [14]. This augmentation in the rate of heat transfer is attributed to the sharpening of the temperature gradients in the vicinity of a particle due to the confining walls. For instance, Kung and Harriott [15] reported enhancement in heat transfer in suspensions of fine quartz sand or spherical polymer particles in agitated vessels. Similarly, Ku et al. [16], Ozbelge [17] and Rozenblit et al. [18] have all studied heat transfer to flowing liquid–solid mixtures in horizontal and vertical pipes. While Ku et al. [16] reported augmented heat transfer, Rozenblit et al. [18] concluded that the rate of heat transfer may increase or decrease depending upon the distribution of solids across the cross-section and/or whether there is a bed of solids present in the tube. Thus, a systematic study of heat transfer from a single sphere in tube flow can also serve as a model flow to gain useful insights into the underlying processes for heat transfer to liquid–solid suspensions. In spite of the potential of such an enhancement in the rate of heat transfer, very little information is available on the influence of confining walls on momentum and heat transfer characteristics of a sphere even in Newtonian fluids [19].

On the other hand, many liquids of macromolecular nature (polymers and their solutions) and of multiphase nature (suspensions, emulsions, foams, dispersions, etc.) often display a range of non-Newtonian characteristics including shear-thinning, visco-elasticity and yield stress, etc. [10]. Indeed, it is not uncommon to employ shear-thinning polymer solutions to carry out continuous thermal treatment of food particles in tubes, e.g., see [20–23]. Similarly, non-Newtonian media also form a key constituent of several other processed foods, pharmaceutical products and cleaning agents to improve their homogeneity for a satisfactory end use. Potential advantages of using viscous non-Newtonian carrier media in the context of slurry pipelines have been enumerated elsewhere [10]. Despite their wide occurrence, very little is known about the heat transfer characteristics of a heated sphere immersed in flowing non-Newtonian fluids. Admittedly, many non-Newtonian systems, notably polymeric fluids also exhibit varying levels of visco-elasticity in addition to shear-thinning viscosity, it is, however, reasonable to begin with the simplest (and perhaps the most widely encountered) non-Newtonian characteristics, namely, shear-thinning which is conveniently modeled by the simple power-law model. The level of complexity, in turn, can gradually be built up to accommodate the other type of non-Newtonian features. This work is thus concerned with the prediction of the rate of heat transfer from a heated sphere immersed in the Poiseuille flow of power-law fluids in cylindrical tubes of varying diameters to ascertain the role of blockage on heat transfer. In the first instance, only the symmetric case in which the sphere is mounted at the axis of the cylindrical tube is considered. However, it is deemed useful and instructive first to briefly review the pertinent literature to facilitate the subsequent discussion of the new results obtained in this work.

2. Previous work

Since the bulk of the literature on heat transfer from a sphere in Newtonian fluids has been reviewed elsewhere [5,9,11,23–25,27], it is however fair to say here that the corresponding body of the