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Soret and Dufour effects for three-dimensional flow in a viscoelastic fluid over a stretching surface

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ABSTRACT

This article deals with the Soret and Dufour effects on three-dimensional boundary layer flow of viscoelastic fluid over a stretching surface. The governing partial differential equations are transformed into a dimensionless coupled system of non-linear ordinary differential equations and then solved analytically by the homotopy analysis method (HAM). Graphs are plotted to analyze the variation of different parameters of interest on the velocity, concentration and temperature fields.

1. Introduction

The interest in the flows of non-Newtonian fluids has been increased due to their numerous practical applications in industry and engineering. Such flows widely appear in plastic manufacture, food processing, performance of lubricants, movement of biological fluids, polymer processing, ice and magma flows. In view of the diverse physical structures of such fluids, there is not a single constitutive equation that can govern the flows of all the non-Newtonian fluids. Hence various non-Newtonian fluid models have been introduced in the literature. In general, the classification of these fluids has been presented under three classes namely the differential, the integral and the rate type fluids. The governing equations of these fluids contain various rheological parameters and add extra terms in the resulting differential systems. Therefore to compute either a numerical or analytical solutions to these equations is not an easy task. Many researchers have been recently engaged in analyzing the flows of non-Newtonian fluids [1–10].

Ever since the pioneer work of Sakiadis [11,12], the boundary layer flows generated by a moving surface have been a topic of great interest of the researchers. Such flows are vital in both viscous and the non-Newtonian fluids. The practical applications of such flows are in the manufacturing processes including hot rolling, polymer extension, crystal growing, continuous stretching of hot films, metal spinning etc. [13–20]. The existing literature indicates that much research on stretching flow deals with the mathematical analysis in the two-dimensions. However, three dimensional boundary layer flows past a stretching surface have not been much reported. Ariel [21] found the perturbation and exact solutions for the three-dimensional steady viscous flow past a stretching sheet. Singh [22] examined the three-dimensional flow of a viscous fluid with heat and mass transfer. The series solutions of unsteady three-dimensional MHD viscous flow and heat transfer in the boundary layer flow over an impulsively stretching plate has been obtained by Xu et al. [23].

The purpose of current investigation is to examine the Soret and Dufour effects [24–35] on three dimensional viscoelastic fluid over a stretching surface. Analysis has been presented for the mathematical formulation in Section 2. Section 3 includes the solution of the problem by employing homotopy analysis method (HAM) [36–40]. Sections 4 and 5 report the convergence and discussion of the solution.

2. Mathematical formulation

We consider the three-dimensional flow of an incompressible viscoelastic fluid over a stretching surface at $z = 0$. The motion in fluid is created by a non-conducting stretching surface. The heat and mass transfer characteristics have been considered when both Soret and Dufour effects are present. The continuity, momentum, concentration and energy equations for the present boundary layer flow are reduced to the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \tag{1}$$

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Nomenclature

T_∞, C_∞	ambient temperature and concentration
x, y, z	Cartesian coordinates
c_s, c_p	concentration susceptibility and specific heat
D	diffusion coefficient
a, b	dimensional constants
f	dimensionless velocity
Sh, Nu_x	local Sherwood and local Nusselt number
Re_x	local Reynolds number
T_m	mean fluid temperature
c	stretching rate
Sc, Pr	Schmidt and Prandtl numbers
Sr, Df	Soret and Dufour numbers
K_0	dimensionless viscoelastic parameter
k_T, K_1	thermal diffusion ratio and reaction rate
T, C	temperature and concentration of fluid
k	material fluid parameter
q_w, j_w	wall heat flux, wall mass flux
u, v, w	velocity components
T_w, C_w	wall temperature and concentration
Greeks symbol	
γ	chemical reaction parameter
ϕ	dimensionless concentration
θ	dimensionless temperature
μ	dynamic viscosity
ν	kinematical viscosity
η	similarity variable
α_m	thermal diffusivity

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-k\begin{bmatrix} u\frac{\partial^3 u}{\partial x\partial z^2}+w\frac{\partial^3 u}{\partial z^3} \\ -\left(\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial z^2}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}+2\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2}\right)\end{bmatrix}, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\nu\frac{\partial^2 v}{\partial z^2}-k\begin{bmatrix} v\frac{\partial^3 v}{\partial y\partial z^2}+w\frac{\partial^3 v}{\partial z^3} \\ -\left(\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial z^2}+\frac{\partial u}{\partial z}\frac{\partial^2 w}{\partial z^2}+2\frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y\partial z}+2\frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2}\right)\end{bmatrix}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2}-K_1C+\frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial z^2}, \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha_m\frac{\partial^2 T}{\partial z^2}+\frac{Dk_T}{C_sC_p}\frac{\partial^2 C}{\partial z^2} \tag{5}$$

in which u, v and w are the velocities in the x, y and z directions, respectively, $\nu$the kinematic viscosity, k the material fluid parameter, C the concentration of species, D the coefficients of diffusing species, K_1 the reaction rate, k_T the thermal-diffusion, T the temperature, C_p the specific heat, C_s the concentration susceptibility, α_m the thermal diffusivity and T_m the fluid mean temperature.

The boundary conditions for the present situation can be written as

$$\begin{gathered} u=u_w(x)=ax,\quad v=v_w(y)=by,\quad w=0,\quad T=T_w,\quad C=C_w \quad \text{at } z=0, \\ u\to 0,\quad v\to 0,\quad \frac{\partial u}{\partial z}\to 0,\quad \frac{\partial v}{\partial z}\to 0,\quad T\to T_\infty,\quad C\to C_\infty \quad \text{as } z\to\infty, \end{gathered} \tag{6}$$

where C_w denotes the concentration at the surface, C_∞ is the concentration far away from the sheet, T_w is the surface temperature and T_∞ is the temperature far away from the surface.

Using

$$\begin{gathered} \eta=\sqrt{\frac{a}{\nu}}z,\quad u=axf'(\eta),\quad v=ayg'(\eta),\quad w=-\sqrt{a\nu}\{f(\eta)+g(\eta)\}, \\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \end{gathered} \tag{7}$$

the continuity Eq. (1) is identically satisfied and Eqs. (2)–(6) take the following forms

$$f'''-f'^2+(f+g)f''+K_0\left[(f+g)f^{iv}+(f''-g'')f''-2(f'+g')f'''\right]=0, \tag{8}$$

$$g'''-g'^2+(f+g)g''+K_0\left[(f+g)g^{iv}+(f''-g'')g''-2(f'+g')g'''\right]=0, \tag{9}$$

$$\phi''+Sc(f+g)\phi'-Sc\gamma\phi+ScSr\theta''=0, \tag{10}$$

$$\theta''+\Pr(f+g)\theta'+\Pr Df\phi''=0, \tag{11}$$

$$\begin{gathered} f(0)=0,\ g(0)=0,\ f'(0)=1,\ g'(0)=c,\ \phi(0)=1, \theta(0)=1, \\ f'(\infty)=0,\ g'(\infty)=0,\ f''(\infty)=0,\ g''(\infty)=0,\ \phi(\infty)\to 0,\ \theta(\infty)\to 0, \end{gathered} \tag{12}$$

where $K_0 = ka/\nu$is the dimensionless viscoelastic parameter, prime is the differentiation with respect to η and the constants $a > 0$ and $b > 0$. Furthermore the stretching ratio c, Schmidt number Sc, chemical reaction parameter γ, Prandtl number Pr, Dufour number Df and Soret number Sr are defined as

$$\begin{gathered} c=b/a,\quad Sc=\frac{\nu}{D},\quad \gamma=\frac{K_1}{a},\quad Pr=\nu/\alpha_m, \\ Df=\frac{Dk_T}{C_sC_p}\frac{(C_w-C_\infty)}{(T_w-T_\infty)\upsilon},\quad Sr=\frac{Dk_T}{T_m\nu}\frac{(T_w-T_\infty)}{(C_w-C_\infty)}. \end{gathered} \tag{13}$$

Note that the two-dimensional ($g = 0$) case has been recovered when $c = 0$. For $c = 1$, one finds an axisymmetric case i.e. ($f = g$).

The local Nusselt (Nu_x) and local Sherwood (Sh) numbers can be written as

$$Nu=\frac{xq_w}{k(T_w-T_\infty)},\quad Sh=\frac{xj_w}{D(C_w-C_\infty)} \tag{14}$$

with

$$q_w=-k\left(\frac{\partial T}{\partial z}\right)_{z=0},\quad j_w=-D\left(\frac{\partial C}{\partial z}\right)_{z=0}, \tag{15}$$

where q_w and j_w respectively denote the heat and mass fluxes.

Eq. (14) in dimensionless form becomes

$$Nu_x/\mathrm{Re}_x^{1/2}=-\theta'(0),\quad Sh/\mathrm{Re}_x^{1/2}=-\phi'(0) \tag{16}$$

where $\mathrm{Re}_x = u_wx/\nu$ is the local Reynolds number.

3. Series solutions

3.1. Zeroth-order deformation problems

In order to obtain the HAM solution, the velocity distributions $f(\eta)$, $g(\eta)$, $\phi(\eta)$ and $\theta(\eta)$ in the set of base functions

$$\{\eta^k\exp(-n\eta)|k\geqslant 0,\ n\geqslant 0\} \tag{17}$$