International Journal of Heat and Mass Transfer 55 (2012) 2173–2183

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Fully-developed thermal transport in combined electroosmotic and pressure driven flow of power-law fluids in microchannels

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ARTICLE INFO

Article history:
Received 16 June 2011
Available online 3 January 2012

Keywords:
Electroosmotic flow
Power-law fluids
Microchannel
Convection

ABSTRACT

Thermally fully-developed heat transfer has been analyzed for combined electroosmotic and pressure driven flow of power-law fluid in a microchannel. Analytical expressions for transport parameters are presented in terms of the flow behavior index, the length scale ratio (ratio of Debye length to half channel height), dimensionless pressure gradient, and Joule heating parameter (ratio of Joule heating to surface heat flux). Closed form solutions are obtained for some specific values of the flow behavior index, while numerical solutions are presented for general cases. The results show that the temperature variation across the channel increases with increasing the pressure gradient. To reduce the length scale ratio is found to decrease the temperature variation, particularly for shear-thinning fluids. To increase the Joule heating parameter is to enlarge the temperature variation in the channel, especially for shear-thickening fluids. The Nusselt number can be increased by decreasing the length scale ratio due to the electroosmotic effect. Also, the Nusselt number increases with decreasing the values of flow behavior index and dimensionless pressure gradient.

1. Introduction

Liquid flow and thermal transport in microchannels have received considerable attention in the recent year due to its increasing application in microchannel heat sinks for electronic cooling, microfluidic devices used in MEMS sensors and fluid actuator, as well as Lab-on-a-Chip systems for drug delivery, chemical analysis, and biomedical diagnosis. Fluid delivery is crucial in the microfluidic devices because of the small size of these devices and considerably high operating pressure. Various techniques have been developed for liquid pumping over short distances, such as electrostatic, thermopneumatic, piezoelectric and electroosmotic flow (EOF) pumps. Among them EOF has been favored as a promising means due to its numerous advantages, such as absence of moving parts, much simpler design and easier microfabrication, greater degree of flow control, and suitable for highly viscous situations. The flow physics of this unique mechanism has been described in detail by Probstein [1]. If the length scale ratio is much smaller than unity, the velocity profile is nearly uniform (characterized by plug flow). On the other hand, the velocity profile will be similar to that of classical pressure driven (Poiseuille) flow if the length scale ratio is of the order of unity. Most of the previous studies primarily dealt with Newtonian fluid flow through microchannels to ascertain the electrokinetic effects and transport properties. Some examples can be found in [2–6] and the references therein.

However, study on the transport of non-Newtonian fluid through a microchannel is relatively scant in literature. Common fluids handled in Lab-on-a-Chip based microsystems include whole blood samples, DNA solutions, protein or antibody solutions, buffer solutions, etc. These biofluids usually exhibit non-Newtonian fluid behaviors, and this fact renders Newton's law of viscosity insufficiently to completely describe the constitutive behavior of biofluids. The flow characteristics of non-Newtonian fluid is of high interest in numerous applications such as sample collection, dispensing, detection, mixing, and separation of various biological and chemical species on a microchip. In the analysis the non-Newtonian constitutive behavior needs to be taken into account so as to acquire better understanding of biofluids transport through microchannels and to predict more precisely the performance of microfluidic devices. Therefore, instead of the Navier–Stokes equation, the more general Cauchy momentum equation with appropriate fluid constitutive relations should be used to describe the flow features of non-Newtonian fluids. Recently, several studies have been conducted to explore the electroosmotic flow behavior of non-Newtonian fluids in microchannels. Zhao et al. [7] obtained approximate solutions of the velocity distributions as a function of the flow behavior index based on an approximate scheme for the hyperbolic cosine function. Exact solutions of the velocity distributions were also reported for several special values of the flow behavior index. Tang et al. [8] have applied the lattice Boltzmann

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doi:10.1016/j.ijheatmasstransfer.2011.12.022

Nomenclature

C_f	friction coefficient, Eq. (14)
c_p	fluid specific heat
D_h	hydraulic diameter
E_x	electric field component in the streamwise direction
e	electron charge
f	friction factor, $\tau_w/(\rho u_m^2/2)$
G	ratio of Joule heating to surface heat flux, $H\sigma E_x^2/q_s''$
H	half channel height
h	convective heat transfer coefficient
K	flow consistency constant
k	thermal conductivity
k_B	Boltzmann constant
n	flow behavior index
Nu	Nusselt number, hD_h/k
n_0	bulk ion concentration
q_s''	wall heat flux
Re	Reynolds number, $\rho u_m^{2-n} D_h/K$
T	temperature
U	dimensionless velocity
u	fluid velocity
u_{HS}	generalized Helmholtz–Smoluchowski velocity, Eq. (9)
x	streamwise coordinate
y	cross-stream coordinate
z	valence

Greek symbols

Γ	dimensionless pressure gradient
δ	length scale ratio, λ_D/H
ε	permittivity of the fluid
ζ	wall zeta potential
η	non-dimensional cross-stream coordinate
θ	dimensionless temperature
κ	Debye—Hückel parameter, $(2n_0 z^2 e^2/\varepsilon k_B T)^{1/2}$
λ_D	Debye length
ρ	fluid density
σ	fluid electrical conductivity
ρ_e	net electric charge density
τ_w	wall shear stress
ψ	EDL potential distribution

Subscripts

m	mean
s	surface

method to carry out a numerical study of electroosmotic flow of power-law fluids in microchannels. Berli and Olivares [9] presented a theoretical study on the electrokinetic flow of non-Newtonian fluids through slit and cylindrical microchannels. Based on the concept of the Helmholtz–Smoluchowski velocity, which is widely adopted in the electroosmotic flows of Newtonian fluids, Park and Lee [10] proposed a simple method to determine the volumetric flow rate of viscoelastic electroosmotic flows through microchannels. A comprehensive work on flow of power-law fluids in a slit microchannel was carried out by Vasu and De [11] to examine the effects of flow behavior index, zeta potential and channel dimension on velocity distribution, apparent viscosity, volumetric flow rate and friction coefficient. They also performed an analysis of electroosmotic flow of power-law fluids in a rectangular microchannel at high zeta potential [12]. Combined electro-osmotically and pressure driven flow of power-law fluids in a slit microchannel was studied by Babaie et al. [13]. The Poisson–Boltzmann equation for electrical potential was solved numerically without using Debye—Hückel approximation. They reported that the Poiseuille number is an increasing function of the Debye—Hückel parameter, wall zeta potential, and flow behavior index.

Thermal transport in microchannels in the presence of electroosmosis has also attracted many investigators due to its promising applications in microscale devices, such as the design of microchannel heat sink and in the cooling of microchips. In the work of Maynes and Webb [14] thermally fully developed, electroosmotically generated convective transport has been analyzed for a parallel plate microchannel and circular microtube. For both geometries they presented analytical expressions for the fully developed, dimensionless temperature profile and corresponding Nusselt number. Horiuchi et al. [15] obtained analytical solutions for temperature distributions and heat transfer characteristics for thermally developing electroosmotic flow in two-dimensional straight microchannels. They identified the effects of Joule heating on electroosmotic heat transfer in microchannels for both constant surface temperature and constant wall heat flux thermal boundary conditions. Liechty et al. [16] investigated fully-developed electroosmotic flow and heat transfer in cylindrical microtubes at arbitrary wall zeta potentials. Numerical solutions were reported for a range of electrokinetic radius and Joule heating magnitudes. Chakraborty and Roy [17] presented a theoretical analysis to study the transport behavior of thermally developing and hydrodynamically developed electroosmotic flow of nanofluids in parallel plate microchannels. Closed form expressions for the temperature profiles and the Nusselt number variations are obtained, so as to assess the impact of volume fraction of the dispersed nanoparticles on the convective transport characteristics. It is revealed that the influences of the nanofluids are much more noticeable in the thermal entrance region, as compared to the thermally fully developed region.

Flow inside a microchannel can also be generated by a combination of pressure-driven flow and electroosmotic flow. The flow rate induced by electroosmotic force is usually small, and therefore even a small pressure gradient applied along a microchannel may cause velocity distributions and corresponding flow rates that deviate from the purely electroosmotic flow. The pressure gradient may stem from several reasons, such as the presence of alternative pumping mechanism and the existence of variations in the wall zeta potential. Quite a lot of research works have been conducted to examine heat transfer characteristics of combined electroosmotic and pressure driven microflows (see, for example, [18–26]). When electroosmotic and traditional pressure forces are present simultaneously, the resulting velocity profile is a superposition of the electroosmotic and pressure driven flows. The velocity distributions of the combined flow are much different from those of purely electroosmotic flow or traditional pressure-driven flow. As a result, the thermal behavior alters to a certain extent, depending on the relative importance of the two driving mechanisms. Recently, thermal transport characteristics of combined electro-osmotically and pressure-driven microflows with the Joule heating effect has been analyzed by Chen [24] for both constant and variable fluid properties. The above work was extended to account for viscous heating effects by Sadeghi and Saidi [25] and Dey et al. [26] for constant fluid properties. However, all the aforementioned investigations [18–26] are concerned with Newtonian fluids. Study on the electroosmotic heat transfer of non-Newtonian fluids in microchannels is less focused. Based on an approximation of the hyperbolic sine function, Das and Chakraborty [27] analysed the convective transport characteristics of microchannel flows under the sole influence of electrokinetic