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Technical Note

Numerical simulation of the onset characteristics in a standing wave thermoacoustic engine based on thermodynamic analysis

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ABSTRACT

In this paper, a simplified physical model of standing wave thermoacoustic engines (SWTE) is developed based on thermodynamic analysis. Transient pressure drop and heat transfer data are first calculated based on linear thermoacoustic theory. The effects of stack spacing, charge pressure, and resonator length on onset temperature were investigated and compared with experimental results. The calculations agree well with the experimental results, which validates the model for calculating the onset conditions.

1. Introduction

A thermoacoustic engine converts heat into acoustic power by thermoacoustic effects. Linear thermoacoustic theory, proposed by Rott [1], is now widely used in explaining the conversion from thermal energy to acoustic energy. However, the equations in linear thermoacoustic theory cannot apply to the simulation in time domain, which is indispensable for understanding the mechanism of thermoacoustic conversion. In recent years, nonlinear thermoacoustic theories have been developing rapidly [2–4]. However, there are few methods that supply sufficient and reliable data for the oscillating flow in the stack or regenerator, due to the limited theoretical [5–8] and experimental [9–11] studies. A novel thermodynamic analysis method was proposed by de Waele [12] to reveal the onset conditions of a traveling wave thermoacoustic engine. To simplify the solution, an ideal regenerator was assumed in de Waele's study. However, the method cannot be applied directly to SWTEs due to the intrinsically imperfect heat transfer in the stack of a SWTE.

In this study, all components of a SWTE were converted analogously to build a simplified model based on de Waele's research [12]. In addition, the energy equation in the stack was deliberately introduced to describe the imperfect heat transfer characteristics. The onset temperature difference were calculated and verified by experiments.

2. Theoretical analysis of a SWTE

As shown in Fig. 1(a), a SWTE consists of a hot tube, hot end heat exchanger (HHX), stack, cold end heat exchanger (CHX), resonator, buffer. It is assumed that the pressure amplitude is small, the solid temperature profile in the stack is linear and constant, the void volumes of HHX and CHX are negligible. Only the compliance effect is considered in the hot tube and buffer, and only the inertance effect is considered in the resonator. The SWTE is simplified to the model as shown in Fig. 1(b), in which the resonator is replaced by a resonating piston.

Let m_R be equal to the mass of gas contained in the resonator in front of the piston. To make the frequency characteristic of the resonator the same as that of the piston, it has:

$$L'_R = \frac{1}{3}L_R = \frac{2}{3\pi}L_{ac} \tag{1}$$

$$V_b = V'_R + V'_b \tag{2}$$

where V'_b is the volume of the buffer before the conversion.

Let $T_m = (T_h + T_c)/2$, the pressure in the hot buffer $p_h = p_0 + \delta\, p_h$, the pressure in the cold end $p_c = p_0 + \delta\, p_c$, the pressure and temperature in the stack $p_s = p_0 + \delta\, p_s$, and $T_s = T_0 + \delta\, T_s$.

In adiabatic condition, the volume flow rate U_h is:

$$U_h \approx -\frac{V_h}{\gamma p_0}\frac{d\delta p_h}{dt} \tag{3}$$

The parallel-plate type stack is used in this analysis for its high efficiency [13]. According to the continuity equation of the stack:

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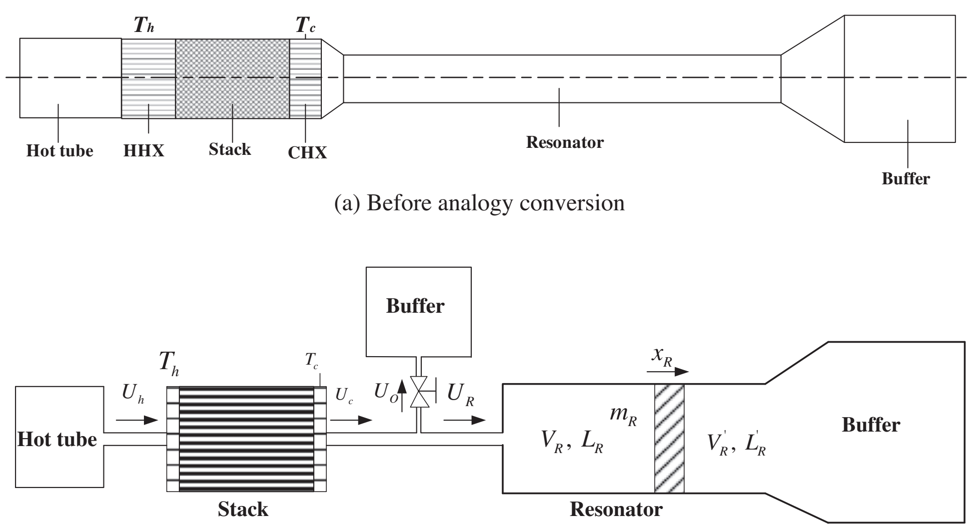
Nomenclature

A	area, (m^2)
c_p	specific heat at constant pressure, (J/(kg K))
C	flow conductance, (m^3/(Pa s))
D	diameter, (m), a coefficient for pressure drop
f	frequency, (Hz)
f_κ	spatial average thermal function
f_v	spatial average viscous function
H	a coefficient for heat transfer
L	length, (m)
m	mass, (kg)
p	pressure, (Pa)
Q	energy source, (W)
T	temperature, (K)
U	volume flow rate, (m^3/s)
V	volume, (m^3)
y_0	half of the stack spacing, (m)
ϕ	porosity of the stack
γ	specific heat ratio
θ_Q	a coefficient for heat transfer
θ_v	a coefficient for pressure drop
ρ	density, (kg/m^3)
ω	angular frequency, (s^{-1})
Prefix	
δ	deviation from average value
Δ	difference value
Subscripts	
b	buffer behind the resonator
c	cold end
g	gas
h	hot end
O	orifice
R	resonator
s	stack



(a) Before analogy conversion

(b) After analogy conversion

Fig. 1. Schematic of a SWTE before and after an analogy conversion.

$$U_h - \frac{T_h}{T_c}U_c = \frac{m_s}{\rho_h}\left(\frac{1}{p_s}\frac{d\delta p_s}{dt} - \frac{1}{T_m}\frac{d\delta T_s}{dt}\right) \quad (4)$$

In time domain, the pressure drop along the stack can be expressed as:

$$\delta p_h - \delta p_s = D_h\left(1 + \theta_{v_h}\frac{\partial}{\partial t}\right)U_h \quad (5)$$

$$\delta p_s - \delta p_c = D_c\left(1 + \theta_{v_c}\frac{\partial}{\partial t}\right)U_c \quad (6)$$

where the coefficients D and θ_v indicate the pressure drop dependence of volume flow rate U and first-order derivation of U, respectively. They can be calculated by using the following complex equation according to the short stack approximation [13]:

$$D(1 + i\omega\theta_v)U = \frac{1}{2}\frac{i\omega\rho_m}{(1 - f_v)}\frac{L_s}{A_g}U \quad (7)$$

where $A_g = \phi\ A_s$ is the gas area of the stack cross section.

Neglecting the small terms of kinetic energy term, the energy equation in the stack can be expressed as:

$$\frac{\partial(c_v m_s T_s)}{\partial t} = \frac{\gamma}{\gamma - 1}p(U_h - U_c) - H\left(1 + \theta_T\frac{\partial}{\partial t}\right)(T_s - T_w) \quad (8)$$

where the last term in the right side represents the heat transfer between solid and the gas in the stack, and the coefficient H and θ_T means the heat transfer dependence of temperature difference ΔT and first-order derivation of ΔT. They can be calculated by using the following complex equation based on Swift's theory [13]:

$$H(1 + i\omega\theta_T) = \rho c_p V_s\frac{i\omega f_\kappa}{1 - f_\kappa} \quad (9)$$

The volume flow rate through the orifice U_O is:

$$U_O = C_O\delta p_c = C'\mathrm{Re}(f_{\kappa_R})^2\delta p_c \quad (10)$$

where the thermal function in resonator f_{κ_R} is used to calculate the flow conductance.

The motion equation of the piston in the resonator yields: