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Analysis of Bejan's heatlines on visualization of heat flow and thermal mixing in tilted square cavities

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ABSTRACT

This article analyzes the detailed heat transfer phenomena during natural convection within tilted square cavities with isothermally cooled walls (BC and DA) and hot wall AB is parallel to the insulated wall CD. A penalty finite element analysis with bi-quadratic elements has been used to investigate the results in terms of streamlines, isotherms and heatlines. The present numerical procedure is performed over a wide range of parameters ($10^3 \leqslant Ra \leqslant 10^5, 0.015 \leqslant Pr \leqslant 1000, 0° \leqslant \varphi \leqslant 90°$). Secondary circulations cells are observed near corner regions of cavity for all φ's at $Pr = 0.015$ with $Ra = 10^5$. Two asymmetric flow circulation cells are found to occupy the entire cavity for $\varphi = 15°$ at $Pr = 0.7$ and $Pr = 1000$ with $Ra = 10^5$. Heatlines indicate that the cavity with inclination angle $\varphi = 15°$ corresponds to large convective heat transfer from the wall AB to wall DA whereas the heat transfer to wall BC is maximum for $\varphi = 75°$. Heat transfer rates along the walls are obtained in terms of local and average Nusselt numbers and they are explained based on gradients of heatfunctions. Average Nusselt number distributions show that heat transfer rate along wall DA is larger for lower inclination angle ($\varphi = 15°$) whereas maximum heat transfer rate along wall BC occur for higher inclination angle ($\varphi = 75°$).

1. Introduction

The phenomenon of natural convection in closed cavities has received considerable attention due to its practical relevance in various applications, such as geophysics simulation, food engineering, room heating and ventilation design, solar energy collection design, cooling of computer systems and other electronic equipments, etc. [1–6]. There are two ways to describe the phenomenon of natural convection such as, experimental methods and numerical simulations. Analysis with numerical methods is more preferable as high cost is involved in experimental methods. The fluid flow and heat transfer characteristics of such systems are predicted by the mass, momentum and energy conservation equations with appropriate boundary conditions. Processes involving natural convection flows within cavities are important for rectangular and nonrectangular enclosures. A number of studies on natural convection within enclosures with various shapes (square, triangular, trapezoidal, etc.) have also been performed and the solutions for both flow and thermal fields have been reported in literature [7–9].

A few earlier investigations on natural convection flows within cavities for various thermal boundary conditions may be outlined as follows. Ganzarolli and Milanez [10] studied natural convection flow in rectangular enclosure heated from below and cooled along a single side or both sides using a streamfunction–vorticity formulation and presented the numerical results for various Rayleigh numbers ($10^3 \leqslant Ra \leqslant 10^7$), Prandtl numbers ($0.7 \leqslant Pr \leqslant 7$) and aspect ratios ($0.66 \leqslant AR \leqslant 8$). Corcione [11] studied numerical investigation on natural convection in air-filled, 2D rectangular enclosures heated from below and cooled from above with wide variety of thermal boundary conditions at the side walls. Ravi et al. [12] studied the structure of steady, laminar natural convection in a square enclosure for high Rayleigh numbers with inclination angle, $\varphi = 90°$. The effect of a simultaneous differential heating of both the horizontal and vertical walls of a square cavity was analyzed by Shiralkar and Tien [13]. November and Nansteel [14] performed analytical and numerical studies on natural convection heat transfer in a square, water filled enclosure heated from below and cooled on one vertical side. The numerical study of natural convection in a square enclosure for hot bottom wall and various hot/cold side walls with insulated top wall was carried out by Roy and Basak [15]. A number of studies on the convection patterns in the square enclosure are carried out by several investigators such as Patterson and Imberger [16], Hyun and Lee [17], Lage and Bejan [18], Xia and Murthy [19] and Nicolette et al. [20].

The brief literature survey as mentioned above, provides various studies of flow and heat transfer characteristics within the

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Nomenclature

g	acceleration due to gravity (m s^{-2})
L	side of the tilted square cavity (m)
N	total number of nodes
n	normal vector to the plane
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
p	pressure (Pa)
P	dimensionless pressure
Pr	Prandtl number
R	residual of weak form
Ra	Rayleigh number
T	temperature of the fluid (K)
T_h	temperature of hot wall (K)
T_c	temperature of cold wall (K)
u	x component of velocity (m s^{-1})
U	x component of dimensionless velocity
v	y component of velocity (m s^{-1})
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
x	distance along x coordinate (m)
Y	dimensionless distance along y coordinate
y	distance along y coordinate (m)

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
β	volume expansion coefficient (K^{-1})
γ	penalty parameter
Γ	boundary of two dimensional domain
θ	dimensionless temperature
ν	kinematic viscosity (m^2 s^{-1})
ρ	density (kg m^{-3})
φ	inclination angle with the positive direction of X axis
Φ	basis functions
ψ	dimensionless streamfunction
Π	dimensionless heatfunction
ϵ	error in heat balance within the cavity

Subscripts

i	residual number
k	node number

square cavity. Due to various applications of natural convection within tilted cavity such as, solar energy collectors, solar passive design, double-glazed windows, car batteries, cooling of electronic equipment and industrial processes such as crystal growth and electrochemical metal refining, the heat transfer by natural convection within the tilted cavity in passive solar systems constitutes a major area of recent researches. It is the recognition of these effects on the performance of passive systems that necessitates to investigate the heat energy flow within tilted square cavity to design of efficient thermal systems. During the past two decades, few experimental and numerical studies have been carried out for understanding fluid flow and associated heat transfer during natural convection within tilted cavities. These studies were focused to understand the effect of various Rayleigh numbers (Ra), Prandtl numbers (Pr) and tilted angles (φs) on fluid flow and heat transfer.

The effect of inclination angle on natural convection within tilted cavities has been discussed by few investigators [21–35]. Baez and Nicolas [21] numerically analyzed the natural convection heat transfer within a rectangular cavity filled with porous media for various inclination angle. Natural convection within tilted square cavity with one side cooled at uniform temperature and the partially heated at the opposite wall is investigated by Corcione and Habib [22]. Experimental and numerical values of the Nusselt number for heat transfer in a tilted square cavity with uniform temperature on one inclined side and at a lower uniform temperature on the opposite side were studied by Ozoe et al. [23]. Hamady et al. [24] analyzed the effect of inclination angle on the heat transfer characteristics in an air-filled differentially heated enclosure. They found the strong dependence of the heat flux on the inclination angles (φs) and Rayleigh numbers (Ra). Bairi et al. [25] investigated numerically and experimentally the natural convection in rectangular inclined enclosures for high Rayleigh numbers. They obtained a correlation between Nusselt numbers and Rayleigh numbers and they found the minimal value for Nusselt numbers at inclination angle, $\varphi = 270°$. Rasoul and Prinos [26] studied the effect of inclination angle on laminar natural convection in a square cavity for inclination angles ($40° \leqslant \varphi \leqslant 160°$), Rayleigh numbers ($10^3 \leqslant Ra \leqslant 10^6$) and Prandtl numbers ($0.02 \leqslant Pr \leqslant 4000$). Schinkel and Hoogendoorn [27] carried out numerical and experimental study on natural convection within inclined air filled cavities with aspect ratios ($1 \leqslant AR \leqslant 18$) at Rayleigh numbers ($Ra$) varying within $2 \times 10^4 - 5 \times 10^5$ and tilting angles of the enclosure ($40° \leqslant \varphi \leqslant 90°$). Cianfrini et al. [28] studied natural convection in tilted square enclosures with two adjacent walls heated and the two opposite walls cooled. They have presented flow patterns and temperature distributions with Rayleigh numbers ($10^4 \leqslant Ra \leqslant 10^6$) and inclination angles of cavity ($0° \leqslant \varphi \leqslant 360°$). Oztop [29] numerically investigated the natural convection in a partially cooled and inclined rectangular enclosure filled with saturated porous medium. It is found that the inclination angle is dominant parameter on heat transfer and flow circulations. Aydin et al. [30], Aounallah et al. [31] and Lee and Lin [32] made different applications for fluid filled inclined enclosures and found that inclination angle can be used as a control parameter of heat transfer and fluid flow. The effect of inclination angle on natural convection is also analyzed by Catton et al. [33], Strada and Heinrich [34], Dropkin and Somercales [35] for different Rayleigh numbers based on arbitrary angle of inclinations.

The detailed analysis on heat flow via heatlines is important for various tilted cavities. A generalized formulation on heatlines for any tilt angle is developed for the first time in this work. Also the effect of inclination of the cavity to the direction of gravity has many engineering applications. In the case of tilted cavity, enclosures are inclined to the direction of gravity. Hence, buoyancy forces have both components relative to the walls of the enclosure which influence strongly the flow structure and the heat transfer therein. The aim of the present investigation is to analyze the natural convection flow within the tilted square cavity for hot wall AB and cold side walls (BC and DA) in the presence of insulated wall CD with tilting angles of the enclosure ($0° \leqslant \varphi \leqslant 90°$). The heating strategy within the inclined cavity also needs the complete understanding of thermal mixing and visualization of heat flow which forms the basis of the present work.

Streamlines and isotherms are generally used for numerical analysis of fluid flow and temperature distribution. Streamlines are useful to visualize the fluid flow whereas isotherms determine only temperature distribution which may not be suitable to visualize the direction and intensity of heat transfer. In order to visualize