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Unsteady MHD dusty viscoelastic fluid Couette flow in an irregular channel with varying mass diffusion

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ABSTRACT

The present investigation deals with the study of unsteady, MHD, heat absorbing and chemically reacting dusty viscoelastic (Walter's liquid model-B) fluid Couette flow between vertical long wavy wall and a parallel flat wall saturated with porous medium subject to convective cooling and varying mass diffusion. The perturbation method is employed to analyze the coupled equations involving nonlinear problem and solution for the velocity, temperature and concentration distributions are obtained analytically. The graphical results are presented and the physical aspects are discussed in detail to interpret the effect of various significant parameters of the problem. The effect of the skin-friction, rate of heat and mass transfer coefficients at the channel walls are tabulated.

1. Introduction

The use of non-Newtonian fluids has grown considerably because of more applications in chemical process industries, food preservation techniques, petroleum production and power engineering. The flow and heat transfer behavior of viscoelastic fluid flow between parallel plates is of special interest in many engineering fields. In view of these applications, the study of boundary layer behavior has been channelized to viscoelastic fluids. Beard and Walter [1] had introduced the boundary layer treatment for an idealized viscoelastic fluid. The heat transfer in the forced convection flow of a viscoelastic fluid of Walter model was investigated by Rajagopal [2]. Some theoretical studies [3–5] had analyzed the flow and heat transfer characteristics of Walter's Liquid Model-B. The problem of MHD flows has wide range of applications in emerging fields due to an electro-magnetic field, are relevant to many practical applications in Geophysical and Astrophysical situations, the metallurgy industry, cooling of continuous strips and filaments drawn through a quiescent fluid. Makinde and Chinyoka [6] had analyzed the MHD transient flows and heat transfer of dusty fluid in a channel. The influence of magnetic field on the Couette flow was investigated in some studies [7–9]. Convective flow with simultaneous heat and mass transfer under the influence of a magnetic field and chemical reaction have attracted a considerable attention of researchers because such process exist in many branches of science and technology. Mass transfer processes containing a fluid that flows under laminar regime in a Couette fashion has a variety of applications in processes related to chemical engineering. Some of these applications include solid–liquid extraction, coating processes and some important biochemical system such as mass transfer processes in dialyzers, oxygenators, and other membrane processes. Some investigations regarding the effect of chemically reacting fluid flows are mentioned in the studies [10–13]. In literature, the attention has been devoted to the case of Couette flow during the last few decades [14–18].

The hot walls and the working fluid are usually emitting the thermal radiation within the systems. Goulard and Goulard [19] studied the coupling between convective and radiative heat transfer for one dimensional Couette flow. The role of thermal radiation is of major importance in the design of many advanced energy convection systems operating at high temperature and knowledge of radiative heat transfer becomes very important in the nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles and space vehicles [20–24]. The main advantage of Joule heating is the rapid and relatively uniform heating achieved together with the lower capital cost compared to other electro heating methods such as microwave and radio frequency heating. The applications of Joule heating technique in industries include the blanching, evaporation, dehydration and pasteurization of food products. Some recent interesting contributions pertaining to heat transfer aspects of Joule heating are cited in Refs. [25–27]. It is well-known that the coupling between the transport of heat and mass takes place because the density of the fluid mixture depends

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Nomenclature

X	dimensional coordinate along the channel
Y	dimensional coordinate perpendicular to the channel
U	dimensional dusty fluid velocity along X direction
V	dimensional dust particles velocity along X direction
d	dimensional width of the channel
t^*	dimensional time
g	gravitational acceleration
K_0	dimensional viscoelastic parameter
N_0	density of the dust phase
K_1	stokes resistance
m	magnetic field parameter
K^*	dimensional porous permeability parameter
B_0	strength of the magnetic field
T	dimensional temperature of the fluid
T_1	wavy wall temperature
T_2	flat wall temperature ($T_2 > T_1$)
$\overline{T}$	mean value of T_1 and T_2
k	coefficient of thermal conductivity
Q_T	dimensional heat absorption coefficient
C_P	specific heat at constant pressure
q_r	dimensional radiative heat flux
$K_{\lambda_1 w}$	radiation absorption coefficient at the wall
$e_{b\lambda_1}$	Plank's function
K_T	thermal diffusion ratio
C	dimensional concentration
C_1	wavy wall concentration
C_2	flat wall concentration ($C_2 > C_1$)
D	molecular diffusivity
K_R	dimensional chemical reaction parameter
n,A	positive real constants
U_P	dimensional velocity of the moving plate
h_f	heat transfer coefficient
Nu^*	dimensional Nusselt number
Sh^*	dimensional Sherwood number
E	viscoelastic parameter
w	relaxation time parameter for dust particles
K	permeability coefficient of porous medium
G_r	thermal Grashof number
G_c	solutal Grashof number
M^2	Hartmann number
P_r	Prandtl number
F	thermal radiation parameter
E_c	Eckert number
S_c	Schmidt number
K_r	chemical reaction parameter
S_r	Soret number
u_p	velocity of the moving plate
B_i	Biot number
Nu	Nusselt number
Sh	Sherwood number

Greek symbols

ρ	density
μ	dynamic viscosity
ν	kinematic viscosity
σ_e	electric conductivity of the fluid
β_T	thermal expansion coefficient
β_C	concentration expansion coefficient
λ_1	frequency parameter
δ	mass concentration of dust particles
α_T	heat absorption parameter
λ	frequency parameter of the wavy wall
ϵ	amplitude parameter of the wavy wall
ε	small positive constant ($\varepsilon \ll 1$)
τ_f^*	dimensional shear stress of dusty fluid
τ_p^*	dimensional shear stress of dust particles
τ_f	skin friction of the dusty fluid
τ_p	skin friction of the dust particles

Subscripts

f	fluid
p	particle

on both temperature and concentration. The mass fluxes can also be created by temperature gradients and this is called as Soret (thermal diffusion) effect. In many studies Soret effect is neglected, on the basis that it is smaller order of magnitude than the effects described by Fourier's and Fick's laws but it is important in the fields such as Geophysical systems, chemical process engineering and high speed aerodynamics. Due to the importance of Soret effect for the fluids with light as well as medium molecular weight, [28–30] had studied and reported the significance of Soret effect. Fluid flow over wavy boundaries can be observed in several natural phenomena, viz., the generation of wind waves on water, formation of sedimentary ripples in river channels and dunes in the desert. The analysis of such flows finds applications in different areas, such as transpiration cooling of reentry vehicles and rocket boosters, cross-hatching on ablative surfaces, and film vaporization in combustion chambers. Very few investigations in literature had been made to study the problems involving fluid flow in wavy walled channel [31–33]. The convective boundary condition is more general and realistic especially with respect to several engineering and industrial processes like transpiration cooling process and material drying. In recent investigations, some researchers [34–36] have shown interest in obtaining self-similar solutions of boundary layer flows over flat surfaces with convective boundary conditions.

To the best of author's knowledge, the influence of free convective heat and mass transfer analysis on Couette flow with time dependent convective cooling and varying mass diffusion has not been studied before. Therefore the main goal here is to construct a mathematical model to examine the influence of heat absorption, radiation, Joule heating, viscous dissipation, chemical reaction and thermal diffusion on unsteady MHD Couette flow of a dusty viscoelastic fluid in a vertical irregular porous channel with convective boundary and varying mass diffusion. The results of parametric study on the velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number distributions are shown graphically and the physical aspects are discussed. The results are expected to be applicable to realistic engineering situations cited earlier. The organization of the paper is as follows: Problem is formulated in Section 2. Section 3 includes the solutions for the problem by using perturbation method. Numerical results and discussion are given in Section 4. The conclusions have been summarized in Section 5.

2. Mathematical formulation

A unsteady two-dimensional flow of an incompressible, electrically conducting, dusty viscoelastic fluid (Walter's liquid-B model) Couette flow in an vertical irregular porous channel with convective cooling and varying mass diffusion is shown in Fig. 1.