International Journal of Heat and Mass Transfer 55 (2012) 3100–3113

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Connecting dispersion models and wall temperature prediction for laminar and turbulent flows in channels

O. Grégoire [a,*], M. Drouin [b], O. Simonin [c]

[a] STXN, 25 rue Leblanc, 75015 Paris, France
[b] CEA Saclay, DEN/DANS/DM2S/SFME/LETR, 91191 Gif-sur-Yvette, France
[c] Université de Toulouse, CNRS; IMFT; Allée du Professeur Camille Soula, 31400 Toulouse, France

ARTICLE INFO

Article history:
Received 10 January 2012
Accepted 6 February 2012
Available online 22 March 2012

Keywords:
Heat transfer
Taylor dispersion
Double averaging
Turbulence

ABSTRACT

In a former paper, Drouin et al. [6] proposed a model for dispersion phenomena in heated channels that works for both laminar and turbulent regimes. This model, derived according to the double averaging procedure, leads to satisfactory predictions of mean temperature. In order to derive dispersion coefficients, the so called "closure problem" was solved, which gave us access to the temperature deviation at sub filter scale. We now propose to capitalize on this useful information in order to connect dispersion modeling to wall temperature prediction. As a first step, we use the temperature deviation modeling in order to connect wall to mean temperatures within the asymptotic limit of well established pipe flows. Since temperature in wall vicinity is mostly controlled by boundary conditions, it might evolve according to different time and length scales than averaged temperature. Hence, this asymptotic limit provides poor prediction of wall temperature when flow conditions encounter fast transients and stiff heat flux gradients. To overcome this limitation we derive a transport equation for temperature deviation $(T_w - \langle\overline{T}_f\rangle_f)$. The resulting two-temperature model is then compared with fine scale simulations used as reference results. Wall temperature predictions are found to be in good agreement for various Prandtl and Reynolds numbers, from laminar to fully turbulent regimes and improvement with respect to classical models is noticeable.

1. Introduction

Design, optimization and safety analysis of large heating devices such as heat exchangers or nuclear reactor cores are major concerns for many engineers. Those studies rely heavily upon flows and heat exchanges modeling. Indeed, considering the geometrical complexity and the size of such systems, it is not possible, to calculate the details of velocity and temperature profiles in each sub-channel. However, the primary interest for industrial purpose is not the details of the flow, but rather the description on a large scale of mean flow quantities and heat transfer properties. Such a macroscopic description may be obtained by applying up-scaling methods [9,15,20,21]. Doing so, a nuclear reactor core, for instance, can be described in an homogenized way (porous approach) by means of a spatial filter [1,19]. This averaging procedure leads to modified equations for mean flow variables, with additional contributions that account for small scale phenomena, mainly boundary layers interactions with solids. Actually, such heating devices might be seen as spatially periodic and anisotropic porous media, with the additional difficulty that flows may achieve any regime, from laminar to highly turbulent, within the pores.

In a former article [6], a complete macroscopic mean temperature model for flows in stratified porous media has been presented. Correlations for scalar and temperature dispersion modeling in rectangular, circular and annular pipes have been established and assessed thanks to comparisons with fine scale simulation results. Now, we focus on wall temperature modeling, or, in other words, on heat exchange modeling. The most classical heat exchange coefficients do not account for transients flows or non uniform heat fluxes since they are based upon the assumption that flows are fully established. It is well known, for instance, that heat exchange in pipes inlet region are poorly predicted and *ad hoc* modifications of models are generally used [10]. During a fast transient or when heat fluxes encounter large gradients, flows cannot be considered established anymore. Boundary layers and bulk flow do not react simultaneously to those strong perturbations and are, in a sense, out of phase. Since dispersion modeling relies on the analysis of spatial deviations of flow quantities (the so-called "closure problem") [2], it appears to be a natural way to account for those unbalances.

In this work, we propose to connect dispersion modeling to heat transfer and averaged wall temperature modeling for forced

* Corresponding author. Tel.: +33 164502743; fax: +33 164501307.
E-mail address: olivier.gregoire@cea.fr (O. Grégoire).

0017-9310/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2012.02.011

Nomenclature

A_f	interface between solid and fluid phases (m^2)
$\mathcal{D}^A$	thermal active dispersion vector (m)
$\mathcal{D}^P$	thermal passive dispersion tensor ($m^2\ s^{-1}$)
C_p	Specific heat capacity ($J\ kg^{-1}\ K^{-1}$)
D_h	hydraulic diameter of the pores (m)
$e_{1,2}$	thickness of the central and near-wall layers
f_w	friction coefficient
n_i	ith component of the interface normal vector, pointing towards the solid phase
Pe	Péclet number ($U\ D_h/\alpha_f = Re\,Pr$)
Pr	Prandtl number (ν_f/α_f)
Pr_t	turbulent Prandtl number (ν_t/α_t)
Re	Reynolds number ($U\ D_h/\nu_f$)
R	radius of the pipe
REV	Representative Elementary Volume
Ω	volume of a REV
Ω_1	volume of the central region of the REV
Ω_2	volume of the near-wall region of the REV
T_f	fluid temperature
$T_{1,2}$	fluid temperature averaged over central and wall regions, respectively
$u_{1,2}$	fluid velocity averaged over central and wall regions, respectively
u_τ	friction velocity ($m\ s^{-1}$)
Greek symbols	
α_f	thermal diffusivity of the fluid ($m^2\ s^{-1}$)
α_t	turbulent thermal diffusivity ($m^2\ s^{-1}$)
α_{t_ϕ}	macroscopic turbulent thermal diffusivity ($m^2\ s^{-1}$)
λ_f	thermal conductivity of the fluid (W/m/K)
δ_w	Dirac delta function associated to the walls (m^{-1})
ΔT	wall to mean temperature gap (K)
ΔV	representative elementary volume (REV) (m^3)
ΔV_f	fluid volume included in the REV (m^3)
ζ	active dispersion function (s)
η_j	passive dispersion function (m)
ν_f	kinematic viscosity of the fluid ($m^2\ s^{-1}$)
ν_t	turbulent kinematic viscosity ($m^2\ s^{-1}$)
ρ	density of the fluid ($kg\ m^{-3}$)
ϕ	porosity
Φ	wall heat flux
Σ	solid surface
$\Sigma_{2,1}$	surface delimiting inner and outer region
Other symbols	
$\bar{\cdot}$	statistical average
$\cdot'$	fluctuation from the statistical average
$\langle\rangle$	volume average
$\langle\rangle_f$	fluid volume average
$\langle\rangle_{1,2}$	fluid volume average over central and wall regions of the flow, respectively
$\delta\cdot$	deviation from the fluid volume average
$\cdot^*$	dimensionless quantity
$\cdot_f$	fluid
$\cdot_B$	bulk
$\cdot_w$	wall
$\cdot_t$	turbulent

convection flows in pipes. In Section 2, the averaging procedure and the derivation and closure of the macroscopic mean temperature equation are recalled. In Section 3, we show how it is possible to connect the temperature deviation modeling embedded in porous media approach with classical heat exchange models. Since temperature in wall vicinity is mostly controlled by boundary conditions, it might evolve according to different time and length scales than averaged temperature. Hence the resulting algebraic closure must be seen as an asymptotic limit consistent with both porous media modeling and classical heat exchange modeling for smooth flows.

For flows exhibiting large or rapid variations of boundary conditions, thermal unbalance might reach a high level so other time and length scales have to be taken into account. To overcome the limitation of algebraic models to represent such an effect, a method to derive a balance equation for wall temperature model is exposed in Section 4. New contributions in this equation are closed consistently with the algebraic limit of the porous model presented in Section 3. The resulting two-temperature model (averaged and wall temperatures) is finally assessed by detailed comparisons with fine scale simulation results and classical heat exchange models.

2. Macroscopic temperature equation

We recall in this section the averaging procedure and the derivation and closure of the macroscopic mean temperature equation. Since flows that are considered in this work can be turbulent, a statistical average operator, denoted "$\bar{\cdot}$", is used to handle the random character of turbulence. Our aim is to develop a spatially homogenized modeling of these flows, so we also apply a spatial filter. The spatial average operator used to derive a macroscale model is denoted "$\langle\cdot\rangle_f$". For each average, any quantity ξ may be split into mean and fluctuating components as

$$\xi = \bar{\xi} + \xi' = \langle\xi\rangle_f + \delta\xi, \tag{1}$$

and one can write

$$\xi = \langle\bar{\xi}\rangle_f + \langle\xi'\rangle_f + \delta\bar{\xi} + \delta\xi'. \tag{2}$$

Statistical and spatial average properties are summarized in Drouin et al. [6]. The order of application for those averages is discussed in Pinson et al. [13].

In this study, incompressible and undilatable, single phase flows in saturated, rigid porous media are considered. Fluid properties (density, viscosity, heat capacity) and the porosity of the medium are assumed constant. Finally, we shall assume that thermal interactions with solids reduce to an external forcing for the fluid temperature. Under those hypothesis, macroscopic conservation of mass equation reads [13]:

$$\frac{\partial\langle\bar{u}_i\rangle_f}{\partial x_i} = 0. \tag{3}$$

Considering the thermal boundary condition on the wall $\mathcal{A}_f$ for statistically averaged temperature

$$\alpha_f\frac{\partial\bar{T}_f}{\partial x_i}\,n_i = \frac{\bar{\Phi}}{(\rho C_p)_f} \quad \text{on } \mathcal{A}_f \tag{4}$$

and under the first gradient approximation for turbulent heat flux:

$$\overline{u_i'T_f'} = -\alpha_t\frac{\partial\bar{T}_f}{\partial x_i}, \tag{5}$$

Drouin et al. [6] (see Section 3) derived the following equation for mean temperature: