International Journal of Heat and Mass Transfer 55 (2012) 3295–3306

Contents lists available at SciVerse ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# Effects of wall roughness and velocity slip on streaming potential of microchannels

H.M. Park *, H.D. Lee

*Department of Chemical and Biomolecular Engineering, Sogang University, Seoul, Republic of Korea*

A R T I C L E  I N F O

*Article history:*
Received 30 June 2011
Received in revised form 6 February 2012
Accepted 6 February 2012
Available online 28 March 2012

*Keywords:*
Electrokinetic flow
Wall roughness
Velocity slip

A B S T R A C T

Microchannels made of polymeric materials usually have wall velocity slip. Sometimes, these microchannels are not smooth. Wall roughness is introduced by manufacturing processes or caused by adhesion of biological particles from the liquids. For a proper design and operation of these microchannels, it is important to secure an accurate value of zeta potential, which determines the volumetric flow rate of electroosmotic flows. Zeta potential of microchannels is usually determined by measuring the streaming potential. In the present investigation, we have derived a simple formula that predicts accurately the streaming potential of microchannels having wall roughness and Navier velocity slip, and investigated how the streaming potential is affected by wall roughness, slip coefficient, zeta potential and bulk ionic concentration. The simple formula, which requires numerical solution of three sets of Stokes equation sequentially using a small grid number, yields very accurate results as compared with the numerical solution of nonlinear partial differential equations employing a very large number of grids. It is found that the streaming potential per pressure drop increases with respect to slip coefficient and decreases with respect to wall roughness hight. On the other hand, the streaming potential per volumetric flow rate increases as the wall roughness increases.



## 1. Introduction

When an electrolyte solution is forced to flow through a microchannel under a pressure gradient, the ionic charges in the electric double layer are convected in the flow direction and induce the streaming current [1]. If there is no external electric circuit connecting the inlet and outlet of the microchannel, the accumulation of charge sets up an electric field called the streaming potential. The streaming potential has been exploited in sea water desalination. Recently, there is an attempt of using the principle of streaming potential to convert the mechanical energy to the electric energy without producing any carbon dioxide. Yang et al. [2] demonstrated that electricity can be generated by forcing water through a ceramic rod. Besides these exploitations of streaming potential, it has been a principal tool of determining the zeta potential of microchannels [3]. Recently, many polymeric materials have been used for the microfluidic devices, because a variety of fabrication techniques are available for polymeric materials as well as they have a wide range of mechanical and electroosmotic properties [4]. Polymer surfaces are usually hydrophobic and velocity slip occurs at the wall of microchannels made of polymers. Since streaming potential is caused by transport of ions near the wall, it is apparently affected by the velocity slip [5,6]. Another factor affecting the streaming potential is wall roughness. The microchannel surface may exhibit certain degree of roughness introduced by manufacturing techniques or by adhesion of biological particles from the liquids [7]. Sometimes, surface roughness is introduced intentionally to increase the surface area to enhance biochemical reactions. The effects of wall wettability, roughness, interfacial velocity and temperature slip on the thin liquid film flow and evaporation in microchannels are discussed in Ojha et al. [8] and Zhao et al. [9]. In the present investigation, we examine the effects of velocity slip and wall roughness on the streaming potential using a numerical model.

## 2. Electrokinetic model

A two-dimensional rough microchannel made of hydrophobic material is formed by two parallel surfaces with rectangular blocks as shown in Fig. 1a. These rectangular blocks model the wall roughness. The rough channel of Fig. 1a consists of a unit channel, depicted in Fig. 1b, connected periodically. For simple electrolytes that dissociate into two equally charged ions of valence $z_e$ and $-z_e$, the governing equations for steady state electrokinetic flows at very low Reynolds numbers may be written in dimensionless variables as follows [10].

$$\nabla p = \frac{1}{Re}\nabla^2 \mathbf{v} + 2\delta \sinh(\alpha\psi)\nabla\phi \tag{1}$$

$$\nabla \cdot \mathbf{v} = 0 \tag{2}$$

$$\nabla^2 \psi = \beta \sinh(\alpha\psi) \tag{3}$$

$$\nabla^2 \phi = 0 \tag{4}$$

* Corresponding author. Tel.: +82 2 705 8482.
*E-mail address:* hmpark@sogang.ac.kr (H.M. Park).

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doi:10.1016/j.ijheatmasstransfer.2012.02.039

**Nomenclature**

| | |
|---|---|
| $b$ | slip coefficient |
| $e$ | elementary charge |
| $\boldsymbol{f}$ | vector defined in Eq. (14) |
| $g$ | function defined in Eq. (19) |
| $k_B$ | Boltzmann constant |
| $L$ | characteristic length |
| $n_0$ | bulk ionic concentration |
| $p$ | pressure |
| $Q$ | volumetric flow rate |
| $Re$ | Reynolds number |
| $Sc$ | Schmidt number |
| $t$ | time |
| $U$ | characteristic velocity |
| $\mathbf{v}$ | velocity vector |
| $Z_e$ | ion valence |
| *Greek symbols* | |
| $\alpha$ | parameter defined in Eq. (5) |
| $\beta$ | parameter defined in Eq. (5) |
| $\delta$ | parameter defined in Eq. (5) |
| $\delta p$ | pressure drop across a unit channel |
| $\delta\phi$ | electric field drop across a unit channel |
| $\epsilon_0$ | permittivity of vacuum |
| $\epsilon$ | dielectric constant |
| $\zeta$ | zeta potential |
| $\kappa$ | Debye length |
| $\eta$ | normal distance to the wall |
| $\rho_f$ | fluid density |
| $\phi$ | external electric potential |
| $\psi$ | induced electric potential |
| *Superscript* | |
| $e$ | electroosmotic |
| $eof$ | electroosmotic |
| $ext$ | external field |
| $p$ | pressure-driven |
| $r$ | reference field governed by a Laplace equation |
| $str$ | streaming potential |
| $*$ | dimensional variables |

where the dimensionless variables and dimensionless groups are defined using the dimensional variables denoted with superscript asterisk as follows:

$$\nabla = L\nabla^*, \quad \psi = \frac{\psi^*}{\zeta_0}, \quad \mathbf{v} = \frac{\mathbf{v}^*}{U}, \quad p = \frac{p^*}{\rho_f U^2}, \quad t = \frac{t^*}{L/U},$$
$$\phi = \frac{\phi^*}{\zeta_0}, \quad \kappa = \sqrt{\frac{\varepsilon_0 \varepsilon k_B T}{2n_0 e^2 Z_e^2}}, \quad \alpha = \frac{eZ_e\zeta_0}{k_B T}, \quad \beta = \frac{L^2 2n_0 e Z_e}{\varepsilon_0\varepsilon\zeta_0},$$
$$\delta = \frac{Z_e e n_0 \zeta_0}{\rho_f U^2}, Re = \frac{\rho_f L U}{\mu}, \quad b = \frac{b^*}{L} \tag{5}$$

Here, $\psi^*$ is the induced electric potential, $p^*$ the pressure field, $\phi^*$ the external potential imposed on the system or the streaming potential, $n_0$ the bulk ionic concentration, $e$ the elementary charge, $k_B$ the Boltzmann constant, $T$ temperature, $\zeta_0$ the reference zeta potential, $L$ the width of the microchannel, $U$ the characteristic velocity, $\kappa$ the Debye length given by $L/\sqrt{\alpha\beta}$ and $Re$ is the Reynolds number. For typical microchannels with $\zeta_0 = 0.1$ V, $L = 10^{-4}$ m, $n_0 = 10^{-3}$ mol l$^{-1}$ and $U = 8 \times 10^{-4}$ m s$^{-1}$, the dimensionless parameters $\alpha$, $\beta$ and $Re$ have values approximately, $\alpha = 3$–4, $\beta = 10^6$–$10^7$, $Re = 0.05$–0.1 and the Debye length $\kappa$ is less than ten thousandth of the microchannel width. The set of equations (1)–(4) is solved for a periodic domain depicted in Fig. 1b to investigate the effects of wall roughness and velocity slip on the streaming potential. Appropriate boundary conditions are shown in Fig. 1c, where $\eta$ denotes normal direction to the wall, and $\mathbf{n}$ is the unit normal vector and $\mathbf{t}$ the unit tangent vector on the wall. The parameter $b$ is the dimensionless slip coefficient defined by $b = b^*/L$, which is proportional to the magnitude of Navier velocity slip at the wall[5,11].

The flow governed by Eqs. (1)–(4) is driven by pressure gradient and electroosmotic force caused by an electric field. Since the governing equations are a linear set, the flow and pressure fields are decomposed into two parts, each of which is driven by pressure gradient and electroosmotic force, respectively.

$$\mathbf{v} = \mathbf{v}^{(p)} + \mathbf{v}^{(e)}; \quad p = p^{(p)} + p^{(e)} \tag{6}$$

Then, the governing equations for the sets $(\mathbf{v}^{(p)}, p^{(p)})$ and $(\mathbf{v}^{(e)}, p^{(e)})$, representing the pressure-driven flow and the electroosmotic flow, respectively, are found after substituting Eq. (6) into Eqs. (1) and (2):

(Set $p$)

$$\nabla\hat{p}^{(p)} + \delta p \nabla p^{(r)} = \frac{1}{Re}\nabla^2\mathbf{v}^{(p)}; \quad \nabla\cdot\mathbf{v}^{(p)} = 0 \tag{7}$$

(Set $e$)

$$\nabla p^{(e)} = \frac{1}{Re}\nabla^2\mathbf{v}^{(e)} + 2\delta\sinh(\alpha\psi)(\nabla\phi^{(r)})\delta\phi; \quad \nabla\cdot\mathbf{v}^{(e)} = 0 \tag{8}$$

In the derivation of Eqs. (7) and (8), we have set,

$$p^{(p)} = \hat{p}^{(p)} + \delta p p^{(r)}; \quad \phi = \delta\phi\phi^{(r)} \tag{9}$$

where both $p^{(r)}$ and $\phi^{(r)}$ satisfy the Laplace equation with the boundary conditions such that $p^{(r)} = 1.0$ and $\phi^{(r)} = 1.0$ at the left inlet and $p^{(r)} = 0.0$ and $\phi^{(r)} = 0.0$ at the right outlet of the unit channel depicted in Fig. 1c. At the wall, both $p^{(r)}$ and $\phi^{(r)}$ satisfy the boundary condition of zero normal derivatives. Then, $\delta p$ and $\delta\phi$ in Eq. (9) are the actual pressure and potential differences across the unit channel. Because we have separated out the inhomogeneous boundary conditions representing the driving forces, i.e., $\delta p$ and $\delta\phi$, the boundary conditions for $(\mathbf{v}^{(p)}, \hat{p}^{(p)})$ and $(\mathbf{v}^{(e)}, p^{(e)})$ of Eqs. (7) and (8) become periodic at the inlet and outlet and may be represented as depicted in Fig. 2.

Next, Eq. (7) is solved with the boundary conditions shown in Fig. 2a using a regular perturbation technique since the slip coefficient $b$ is very small. Adopting the regular perturbation technique, we can make the slip coefficient $b$ appear explicitly in the final formulas for the streaming potential and volumetric flow rate.

Let

$$\mathbf{v}^{(p)} = \left(\mathbf{v}^{(p)}_{(0)} + b\mathbf{v}^{(p)}_{(1)}\right)\delta p; \quad \hat{p}^{(p)} = \left(\hat{p}^{(p)}_{(0)} + b\hat{p}^{(p)}_{(1)}\right)\delta p \tag{10}$$

Then the set $p$, given by Eq. (7), becomes the following two sets of equations.

(Set $p0$)

$$\nabla\hat{p}^{(p)}_{(0)} + \nabla p^{(r)} = \frac{1}{Re}\nabla^2\mathbf{v}^{(p)}_{(0)}; \quad \nabla\cdot\mathbf{v}^{(p)}_{(0)} = 0 \tag{11}$$