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Discrete-time mixed-integer programming models for short-term scheduling in multipurpose environments

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ABSTRACT

We present new discrete-time mixed-integer linear programming formulations for short-term scheduling in multi-purpose batch plants, the most general sequential production environment. We first discuss how multi-purpose batch plants can be expressed using State-Task Network and Resource-Task Network representations through batch-based definition of states (resources) and tasks. We then develop two models based on each representation that account for limited intermediate storage, and discuss extensions such as limited shared resources and time-varying resource availability/cost. Finally, we present several case studies to illustrate the applicability and performance of the proposed models.

1. Introduction

Production scheduling is a decision-making process that plays an important role in the manufacturing industries. It is concerned with the optimal allocation of shared limited resources (e.g. units, utilities etc.) to production tasks over time (Pinedo, 2012). This type of problems arise in many different types of industries, ranging from small-scale pharmaceutical to large-scale petrochemical processing. Due to its importance, development of mathematical models (Reklaitis, 1996; Pinto and Grossmann, 1998; Kallrath, 2002; Floudas and Lin, 2004; Mendez et al., 2006; Li and Ierapetritou, 2008; Maravelias, 2012; Harjunkoski et al., 2014), as well as algorithm architectures (Pekny, 2002) for solving such problems have been the subject of active research in the field of process systems engineering in the last few decades.

Generally, chemical production scheduling problems can be broadly classified into three categories according to the production environment of the facilities, namely network, sequential, and the combination of the two (i.e. hybrid) (Mendez et al., 2006; Maravelias, 2012). Sequential production environments are similar to discrete manufacturing facilities (e.g. semiconductor manufacturing), where each batch flows through different sectors or stages of the production process. There is a strict material handling restriction according to which each batch has to remain intact throughout the process, i.e. batch mixing or splitting is prohibited. In network production environments, in contrast, handling of materials is flexible, but the plant topology is more complex; the concept of production stages doesn't exist. Hence, different modeling approaches have been adopted for different production environment.

Two frameworks for the development of mathematical programming models for scheduling in network environment were first introduced by Pantelides and coworkers, commonly known as the State-Task Network (STN) and Resourse-Task Network (RTN) representations (Kondili et al., 1993; Shah et al., 1993; Pantelides, 1994; Schilling and Pantelides, 1996).

Scheduling in sequential environments received attention in the late 1980s. An approximate method for scheduling in single-stage facilities was proposed by Musier and Evans (1989), while other researchers focused on developing exact mathematical models (Karimi and Reklaitis, 1985; Pinto and Grossmann, 1995; Cerda et al., 1997; Chen et al., 2002). A more general sequential production environment, commonly termed as flexible flow-shop problem in the OR community, is the multi-stage batch facility. Two different modeling approaches have been proposed to address this problem, namely precedence-based and time-grid-based. Scheduling in multi-purpose facilities, the most general sequential environment, can be viewed as the flexible job-shop problem, where each product goes through a series of product-specific stages, which consist of multi-purpose units that may belong to different stages for different products (Maravelias, 2012; Harjunkoski et al., 2014) (see Fig. 1). Although many studies have been carried out for multi-stage facilities, limited work has been done on scheduling in multi-purpose facilities. Furthermore, existing research on multi-purpose facilities adopt continuous-time representation (Mendez and Cerda, 2003; Ferrer-Nadal et al., 2008), which limits the

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Nomenclature

Sets/indices

$i \in \mathbf{I}$	Batches
$j \in \mathbf{J}$	Units
$k \in \mathbf{K}$	Stages
$r \in \mathbf{R}$	Resources (e.g. utilities, man power)
$t \in \mathbf{T}$	Time points/periods
$\mathbf{I}_{jk}$	Batches on their k^{th} stage that can be processed in unit j
$\mathbf{I}^S_{jk}$	Batches on their k^{th} stage that can be stored in vessel j
$\mathbf{I}_{jkt}$	Batches on their k^{th} stage that can start in unit j at time point t
$\mathbf{J}^P/\mathbf{J}^S$	Processing units/storage vessels
$\mathbf{J}_{ik}$	Process units that can process batch i on stage k
$\mathbf{J}^S_{ik}$	Vessels that can store batch i after being processed on its k^{th} stage
$\mathbf{J}_{ikt}$	Units in which batch i on stage k can start at time point t
$\mathbf{K}_i$	Product route of batch i
$\mathbf{K}^-_{ik}/\mathbf{K}^+_{ik}$	Stages in which batch i has to be processed before/after stage k
$\mathbf{T}_i$	Time periods between due date of batch i and earliest finish time on the last stage
$\mathbf{T}_{ik}$	Time periods in which batch i on stage k can be stored
$\mathbf{T}_{ijk}$	Time points at which batch i on stage k can start in unit j

Decision variables

$X_{ijkt} \in \{0,1\}$	=1 if batch i on stage k starts in unit j at time point t
$X^S_{ijkt} \in \{0,1\}$	=1 if batch i on stage k is being stored in vessel j during time period t
$R_{jt} \in \{0,1\}$	=1 if unit/vessel j is available during time period t
$S_{ikt} \in \{0,1\}$	=1 if batch i that has been processed on stage k is stored during time period t
$U_{rt} \in \mathbb{R}^+$	Total amount of utility r consumed during time period t
$MS \in \mathbb{R}^+$	Makespan

Parameters

α_{ijk}	Fixed cost of processing batch i on stage k in unit j
β_{rt}	Cost of utility r during time period t
γ_i	Inventory cost of the final product of batch i
δ	Discretization length
κ_i	Last stage of the product route of batch i
$\mu_i/\bar{\mu}_i$	Release time for batch i ($\mu_i = \lceil \bar{\mu}_i/\delta \rceil$)
$\tau_{ijk}/\bar{\tau}_{ijk}$	Processing time for batch i on stage k in unit j ($\tau_{ijk} = \lceil \bar{\tau}_{ijk}/\delta \rceil$)
ρ_{ijkr}	Fixed amount of utility r required for processing batch i on stage k in unit j
σ_{ik}	Maximum time in which batch i on stage k can be stored ($\sigma_{ik} = \lfloor \bar{\sigma}_i/\delta \rfloor$)
$\phi_i/\bar{\phi}_i$	Due date for batch i ($\phi_i = \lfloor \bar{\phi}_i/\delta \rfloor$)
ψ_{rt}	Total availability of utility r, during time period t

flexibility in modeling a variety of process features such as time-varying resource availability.

To overcome these limitations, the goal of this work is to develop discrete-time mixed-integer programming (MIP) models for short-term scheduling in multi-purpose batch plant. We also consider intermediate storage between stages. The rest of the paper is structured as follows. In the next section, a detailed review on STN/RTN representations as well as mathematical programming formulations for scheduling in sequential environment are provided. In Section 3, we formally define the problem of interest. In Section 4, we discuss the modeling approaches, and in Section 5 we present the proposed mathematical formulations including some extensions. Finally, several case studies are presented in Section 6 to illustrate the applicability and performance of the proposed models. We use uppercase italic letters for variables, uppercase bold letter for sets, lowercase italic letters for indices, and lowercase Greek letters for parameters.

2. Background

2.1. STN and RTN representations

The STN and RTN representations (Kondili et al., 1993; Pantelides, 1994) enable modeling of complex processes with flexible material handling constraints that are common in chemical industries. The STN representation consists of two types of nodes, namely *state* and *task* nodes. State nodes, denoted by circles, represent materials where a material is defined in terms of its thermochemical properties; i.e. identical materials at different pressure or temperature are treated as different states. Tasks nodes, denoted by rectangles, represent the processing operations which consume and produce one or more states. The first models proposed based on the STN representation adopted a discrete-time representation, where the time horizon is uniformly discretized (Kondili et al., 1993; Shah et al., 1993). These models enforce the following constraints: task-unit allocation, capacity constraint for units and storage vessels, and material balance constraints. It is important to note that a material-based approach is used, i.e. the inventory level of each state is monitored through the material balance constraint. Furthermore, storage of intermediate products in multi-purpose vessels is modeled by introducing additional *storage tasks* performed by the vessels.

The RTN representation offers a unified framework to represent production processes. While STN representation differentiates general resources into materials, units and utilities, the RTN representation considers them in a uniform manner. Specifically, renewable resources such as units and utilities are treated in the same manner as non-renewable resources in that they are consumed by the tasks, but renewable resources are restored after the tasks have completed. The RTN representation consists of two types of nodes: *resource* and *task* nodes, represented by circles and squares, respectively. Resource nodes include states, units and utilities. Task nodes consume and produce one or more resources. The RTN representation leads to models that include fewer types of constraints. Since units are also modeled as resources, one general resource balance constraint is sufficient to ensure feasible task-unit allocation as well as the material balance. The capacity of the units and vessels are enforced by capacity constraint similar to the STN model.

Various continuous-time scheduling models based on the STN and RTN representation were proposed (Schilling and Pantelides, 1996; Ierapetritou and Floudas, 1998; Mockus and Reklaitis, 1999; Giannelos and Georgiadis, 2002), including models that can address process features such as changeover time and various storage policies (Maravelias and Grossmann, 2003b; Sundaramoorthy and

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