Computers and Chemical Engineering xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Constrained Unscented Gaussian Sum Filter for state estimation of nonlinear dynamical systems☆

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ARTICLE INFO

Article history:
Received 26 July 2015
Received in revised form 4 April 2016
Accepted 13 April 2016
Available online xxx

Keywords:
Nonlinear/non-Gaussian systems
Unscented transformation
Constrained Bayesian update
Linear constraints
Optimization
Sum of Gaussians

ABSTRACT

This work presents a novel constrained Bayesian state estimation approach for nonlinear dynamical systems. The proposed approach uses the recently proposed Unscented Gaussian Sum Filter to represent the underlying non-Gaussian densities as sum of Gaussians, and explicitly incorporates constraints on states during the measurement update step. This approach, labeled Constrained-Unscented Gaussian Sum Filter (C-UGSF), can thus model non-Gaussianity in constrained, nonlinear state estimation problems. Its applicability is demonstrated using three nonlinear, constrained state estimation case studies taken from literature, namely, (i) a gas phase batch reactor, (ii) an isothermal batch process, and (iii) a continuous polymerization process. Results demonstrate superior estimation performance along with a significant improvement in computational time when compared to Unscented Recursive Nonlinear Dynamic Data Reconciliation (URNDDR), which is a popular nonlinear, constrained state estimation approach available in literature.

1. Introduction

States of a nonlinear dynamical system evolve based on conservation principles and hence implicitly satisfy physical constraints at any given instant of time. Examples of such constraints include non-negativity of pressure and liquid levels, and mole fractions of multi component mixture belonging to the interval [0 1] with their sum being unity. In recursive Bayesian state estimation, these states are treated as random variables. In particular, by making use of nonlinear stochastic models for the states and measurements, along with available real-time measurements, the aim is to infer the conditional probability densities of the states (Maybeck, 1982). Hence, a practical estimation approach should have the following two features:

(i) nonlinear propagation of the state probability density, and
(ii) incorporation of constraints in the estimation procedure.

Nonlinear propagation of the state probability density function is, in general, an intractable problem. Existing popular approaches in literature, such as Extended Kalman Filter (EKF) (Anderson and Moore, 1979), Unscented Kalman Filter (UKF) (Julier and Uhlmann, 1997, 2004), and Ensemble Kalman Filter (EnKF) (Evensen, 2009), when interpreted within the Bayesian framework, inherently assume that the prior density obtained after propagation is Gaussian. While this assumption makes these three approaches tractable, it may lead to inferior state estimates for scenarios where the propagated prior densities deviate significantly from Gaussianity. On the other extreme, non-parametric approaches such as particle filters (PF) (Arulampalam et al., 2002) do not make any restrictive assumption about the density. However, PFs require a large number of samples or particles, typically several orders of magnitude larger than the number of states, and hence have been applied only to lower dimensional problems (López-Negrete et al., 2011; Rawlings and Bakshi, 2006; Shenoy et al., 2013). Another class of filters, namely, the Gaussian Sum Filters (GS-F) (Sorenson and Alspach, 1971), are based on the premise that a sum of Gaussian densities can represent any probability density function of the states to an arbitrary degree of accuracy (Sorenson and Alspach, 1971; Alspach and Sorenson, 1972; Ito and Xiong, 2000). Examples of such approaches include, Gaussian Sum Extended Kalman Filter (Söderström, 2003), Gaussian Sum Unscented Kalman Filter (GS-UKF) (Šimandl and Duník, 2005), Gaussian Sum Particle Filter

☆ Condensed version of this work has been presented in PSE2015/ESCAPE25.

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http://dx.doi.org/10.1016/j.compchemeng.2016.04.021
0098-1354/

Please cite this article in press as: Kottakki, K.K., et al., Constrained Unscented Gaussian Sum Filter for state estimation of nonlinear dynamical systems. Computers and Chemical Engineering (2016), http://dx.doi.org/10.1016/j.compchemeng.2016.04.021

(Kotecha and Djurić, 2003), and the recently developed Unscented Gaussian Sum Filter (UGSF) (Kottakki et al., 2014). Gaussian sum based approaches require specification of several filter parameters such as number of Gaussians, their underlying moments, and weights assigned to these individual Gaussians. Although GS-Fs eliminate the Gaussianity assumption, they are typically computationally more expensive than the filters that assume Gaussianity. The recently proposed UGSF approach (Kottakki et al., 2014) uses a specific choice of the above parameters which ensures that the computational requirements are similar to UKF while simultaneously retaining the ability to model non-Gaussian densities. Kottakki et al. (2014, 2013) demonstrated the superior performance (as well as computational benefits) of UGSF relative to both UKF and GS-UKF for several case studies.

Presence of constraints on the true states imposes additional challenges over and above the presence of non-linearity and non-Gaussianity. Various extensions of the popular nonlinear state estimation techniques have been proposed in literature to incorporate constraints in the state estimation procedure (Vachhani et al., 2005, 2006; Teixeira et al., 2010, 2009). These include approaches that involve a modification of Bayes' rule by explicitly enforcing constraint satisfaction using numerical optimization. Examples of such approaches are Recursive Nonlinear Dynamic Data Reconciliation (RNDDR^A) (Vachhani et al., 2005), Unscented Recursive Nonlinear Dynamic Data Reconciliation (URNDDR^B) (Vachhani et al., 2006), Modified Unscented Recursive Nonlinear Dynamic Data Reconciliation (MURNDDR^C) (Kadu et al., 2010), and constrained particle filters (López-Negrete et al., 2011; Shao et al., 2010; Prakash et al., 2011), with each approach employing modification of the Bayes' rule based update step in EKF^A, UKF^{B,C} and PF approach, respectively. Approaches that use the standard Bayes' rule update followed by an additional step to impose constraints have also been proposed. Examples include projection based approaches in UKF (Teixeira et al., 2010), density truncation approaches in UKF (Teixeira et al., 2010; Simon, 2010; Kadu et al., 2013) and EnKF (Prakash et al., 2010), and constrained Gaussian sum filters (Straka et al., 2012). While all these approaches ensure constraint satisfaction, the drawbacks associated with the corresponding unconstrained approaches can carry-over to the constrained extensions thereby leading to poor performances or high computational costs.

In this work, we propose to extend UGSF for state estimation of nonlinear dynamical systems in presence of linear inequality constraints. In particular, the update step of UGSF is replaced by a set of constrained optimization problems to ensure that the estimated states are feasible with respect to the linear constraints. As discussed earlier, UGSF does not impose the restrictive Gaussianity assumption on state densities for nonlinear state estimation problems. However, it is applicable only to systems driven by Gaussian process and measurement noises. The computational requirements of UGSF are comparable to UKF and are significantly lower than the computational costs for implementing traditional GS-Fs. It is therefore expected that a constrained extension of UGSF will lead to a constrained state estimation method that will enable modeling of non-Gaussian state densities at reasonable computational costs. However, the proposed filter inherits the limitation of UGSF of being applicable to systems driven by Gaussian process and measurement noises. Further, while the originally proposed UGSF is applicable to nonlinear measurement models (Kottakki et al., 2014), in this work we restrict our attention to systems with linear measurement models. In contrast, the constrained GS-F and constrained PF available in literature (Prakash et al., 2011; Straka et al., 2012; Zhao et al., 2014) are applicable to systems with nonlinear measurement models and non-Gaussian noises, and also incorporate nonlinear constraints (Straka et al., 2012).

Organization of the rest of the paper is as follows: Section 2 discusses the problem statement. Section 3 presents the proposed constrained UGSF approach. Section 4 presents results on case studies selected from literature, and Section 5 concludes the paper.

2. Problem statement

Consider a sampled-data system consisting of nonlinear process dynamics, linear inequality constraints on the states and a linear measurement function as,

$$\mathbf{x}(t_k) = \mathbf{x}(t_{k-1}) + \int_{t_{k-1}}^{t_k} \mathbf{f}(\mathbf{x}(t), \quad \mathbf{u}(t))dt + \mathbf{w}(t_k), \mathbf{w}(t_k) \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \quad (1a)$$

$$\mathbf{A}\mathbf{x}(t_k) \leq \mathbf{b} \quad (1b)$$

$$\mathbf{y}_k = \mathbf{H}\mathbf{x}(t_k) + \mathbf{v}_k, \quad \mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \quad (2)$$

$$\mathbf{x}(t_0) \sim \mathcal{N}(\hat{\mathbf{x}}_{0|0}, \mathbf{P}_{0|0}) \quad (3)$$

where, $\mathbf{x}(t) \in \mathbb{R}^n, \mathbf{u}(t) \in \mathbb{R}^p$, represent the state and input vectors at time t while $\mathbf{y}_k \in \mathbb{R}^m$, $\mathbf{w}(t_k) \in \mathbb{R}^n$, $\mathbf{v}_k \in \mathbb{R}^m$ represent observation, state noise and measurement noise, respectively at time t_k. Further, $\mathbf{w}(t_k)$ and $\mathbf{v}_k$ are assumed to be independent, Gaussian, and white stochastic processes. The initial state is unknown but assumed to have a Gaussian distribution as in Eq. (3). Function $\mathbf{f} : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ in Eq. (1a) represents the nonlinear state dynamics while Eq. (1b) specifies linear inequality constraints on the state vector $\mathbf{x}_k$. It is assumed that the nominal process model obtained from Eq. (1a) by excluding the state noise term, along with the linear inequality constraints (Eq. (1b)) together form a well-posed system, i.e. for feasible values of $\mathbf{x}(t_{k-1})$, the state $\mathbf{x}(t_k)$ obtained through noise-free model is also feasible. Hence,

$$\mathbf{A}\mathbf{x}(t_{k-1}) \leq \mathbf{b} \Rightarrow \mathbf{A}(\mathbf{x}(t_{k-1}) + \int_{t_{k-1}}^{t_k} \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t))dt) \leq \mathbf{b} \quad (4)$$

In Eq. (2), $\mathbf{H} \in \mathbb{R}^{m \times n}$ represents the linear observation model. Measurements $\mathbf{y}_k$ are assumed to be available at regularly spaced sampling instants t_k, $k = 0, 1, 2, 3, \ldots$ with $T_s = t_k - t_{k-1}$ being the sampling interval. For ease of notation, we define $\mathbf{x}_k \triangleq \mathbf{x}(t_k)$. The filtering problem is to find a point estimate for $\mathbf{x}_k$, governed by dynamics in Eq. (1a), using available measurements $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k$ which are related to the states as in Eq. (2), and subjected to linear inequality constraints given by Eq. (1b).

The next section presents the proposed constrained UGSF based solution to this constrained nonlinear filtering problem.

3. Proposed Constrained Unscented Gaussian Sum Filter (C-UGSF) approach for nonlinear state estimation

The basic (unconstrained) UGSF approach as proposed in literature (Kottakki et al., 2014) integrates the idea of unscented transformation with a sum of Gaussian approximation. In particular, UGSF involves an unconstrained selection of sigma points followed by their propagation through the process model. The propagated sigma points are used to synthesize a Gaussian sum approximation of the prior density. Upon availability of measurement, the Gaussian sum prior density is updated using the Bayes' rule to obtain the posterior density. However, constraints on states are not incorporated in this update step. We now propose Constrained Unscented Gaussian Sum Filter (C-UGSF), where the state constraints as given in Eq. (1b) are incorporated in the sigma point