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# Embedded optimization for mixed logical dynamical systems

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Through personal discussions and specifically through his paper with Raman (Raman and Grossmann, 1992), Ignacio Grossmann brought the techniques for translating propositional logic statements into mixed integer inequalities to our attention. These powerful ideas led us to introduce mixed logical dynamical (MLD) systems as a paradigm for describing hybrid systems (Bemporad and Morari, 1999). MLD systems defined our research program for a decade. The senior author (Manfred Morari) is delighted to dedicate this paper to Ignacio in gratitude for his inspirational leadership and wonderful friendship for more than 35 years.

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ABSTRACT

Predictive control of hybrid systems is currently considered prohibitive using embedded computing platforms. To overcome this limitation for mixed logical dynamical systems of small to medium size, we propose to use (1) a standard branch-and-bound approach combined with a fast embedded interior point solver, (2) pre-processing heuristics, run once and offline, to significantly reduce the number of subproblems to be solved, and (3) relaxations of the original MPC problem that allow a trade off between computation time and closed-loop performance. A problem-specific ANSI C implementation of the proposed method can be automatically generated, and has a fixed memory footprint and a code size that is insignificantly larger than that of the subproblem solver. Two extensive numerical studies are presented, where problems with up to 60 binary variables are solved in less than 0.2 s with a performance deterioration of below 2% when compared to an optimal MPC scheme.



## 1. Introduction

Control of hybrid systems is very challenging due to the complex interaction of continuous, possibly switched dynamics with logic. Over the last two decades, receding horizon control (RHC) of mixed logical dynamical (MLD) systems (Bemporad and Morari, 1999) has gained considerable attention, both in research and in industry, as a systematic approach to control synthesis for hybrid systems. The success of this approach, which is often referred to as *hybrid model predictive control* (hybrid MPC), is based on three key factors. First, the MLD framework offers a systematic modeling approach to hybrid systems, and it is suitable to express complex physical processes involving logic. Under some technical assumptions, MLD systems are equivalent to other problem classes from hybrid system modeling such as piece-wise affine (PWA) or linear complementarity systems (Heemels et al., 2001), and the MLD representation allows for an automatic translation into an optimization problem. Second, the ability to achieve constraint satisfaction and high control performance *by design* makes MPC the method of choice for controlling constrained systems. Third, the availability of mature software for building MLD models such as HYSDEL (Torrisi and Bemporad, 2004) and for solving the resulting mixed-integer optimization problems (MIPs) by commercial solvers such as CPLEX (International Business Machines Corp, 2013) enables control engineers to deploy hybrid MPC on

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a *desktop* computing platform. Although available algorithms are not polynomial-time, existing desktop software implementations are usually able to compute a solution in acceptable run-time.

However, existing general purpose MIP solvers have focused on powerful computing platforms, whereas no such solver is currently available for *embedded* computing platforms, limiting the applicability of hybrid MPC to simulation studies or slow dynamical systems with considerable IT infrastructure. Examples are buildings (Guan et al., 2010), or applications in the process industry such as the predictive control of a cement kiln (Stadler et al., 2011).

In this paper, we propose a numerical solver for the class of convex integer multistage problems, into which most hybrid MPC problems can be cast, that enables an implementation on embedded platforms. This is achieved by using a standard branch-and-bound algorithm (Bertsimas and Weismantel, 2005) on top of the recently developed FORCES solver for convex multistage problems (Domahidi et al., 2012). In addition, we propose pre-processing heuristics that can significantly reduce the number of subproblems to be explored during the online solution of the MIP. We demonstrate that the proposed method enables the solution to numerous practical problems, which involve a non-trivial number of discrete and continuous variables, on embedded systems. Furthermore, since the subproblem solver is based on an interior point method, hybrid MPC problems with integer convex *quadratic constraints* can be solved. To the best knowledge of the authors, this is the first such solver that targets embedded systems.

The performance of the proposed method is examined on two extensive numerical examples: (1) the energy management control for fuel cells as an example for switched systems where long prediction horizons are required, and (2) a traffic junction as an example of scheduling and multi-agent switched servicing problems. In all examples, our method achieves computation times in the order of or below those required by CPLEX (International Business Machines Corp, 2013), which is equipped with numerous other heuristics, while being directly implementable on low-cost embedded hardware.

### 1.1. Previous work

Recently, it has been demonstrated by various research groups that solving *convex* optimization problems on embedded computing hardware with iterative methods is feasible at very high sampling rates (Domahidi et al., 2012, 2013; Richter, 2012; Jerez et al., 2013; Kögel and Findeisen, 2011; Ferreau et al., 2008; Frasch et al., 2014). The main tools to achieve this performance are tailored optimization methods, which exploit problem structure, in combination with code generation mechanisms that can automatically generate an implementation for the particular embedded platform and problem (Domahidi, 2012; Mattingley and Boyd, 2012; Ullmann and Richter, 2012; Zometa et al., 2013). Using these techniques, today the majority of convex, linear MPC problems without discrete decision variables admit an implementation even on low-power embedded computing platforms.

In contrast, solving optimization problems arising from hybrid MPC on embedded platforms remains computationally prohibitive. The only exception are small systems with few discrete variables, for which the finite horizon optimal control problem with fixed discrete variables can be pre-solved by means of multi-parametric programming for each admissible initial state (Bemporad et al., 2002). Such problems appear, for example, in power electronics, where computation times of below a millisecond are required, cf. (Mariéthoz et al., 2012; Almer et al., 2013). Even in these applications, one is forced to restrict the prediction and control horizon to one or two time steps due to the combinatorial nature of the problem and the resulting memory requirements, potentially sacrificing the favorable properties of MPC with long horizons in terms of closed-loop performance and constraint satisfaction (Rodriguez et al., 2007).

Existing work on the online solution of MIPs for hybrid MPC (Axehill, 2008) employs dual active set methods as subproblem solvers, which have the advantage of effective warm-starting, as well as dual gradient projection methods, resulting in substantial speedups when compared to general purpose solvers. However, both subproblem solvers are restricted to the class of convex quadratic programs (QPs). Semi-definite programming (SDP) relaxations, which are tighter than the standard relaxations used in this paper, are examined by Axehill et al. (2010), but it is unclear whether the numerical performance and robustness of SDP solvers is competitive in a real-time setting. Kirches (2011), Sager (2005) and Sager et al. (2011) present efficient numerical methods for systems with nonlinear, continuous-time dynamics and discrete states or inputs, applying direct multiple shooting and sequential quadratic programming techniques combined with a branch-and-bound algorithm to solve the mixed-integer nonlinear program very efficiently. A tailored rounding scheme is presented by Sager et al. (2012), for the same setting, giving bounds on the integer approximation error. This is applied to discrete-time systems with combinatorial constraints, in particular switching constraints (Jung et al., 2014).

### 1.2. Contribution

In order to reduce the average computation time for solving the MIP arising from hybrid MPC, we propose to use, in addition to a standard branch-and-bound algorithm (Bertsimas and Weismantel, 2005), two heuristics that can be applied to all integer convex multistage problems, and examine their effect on the closed-loop performance in extensive numerical experiments.

The first heuristic is based on relaxing the multistage optimization problem in all but the first $M < N$ time steps, where $N$ is the control horizon length, either in terms of integer feasibility or in terms of optimality. This idea is motivated by the implementation of MPC in a receding horizon fashion, where at each sampling time only the first input of the sequence of computed inputs is applied to the system. One such relaxation is to allow the binary variables to take continuous values from the unit interval $[0, 1]$. While this generally reduces the worst-case runtime by effectively changing the depth of the branch-and-bound tree, our numerical experiments show that the effect of this partial relaxation on the closed-loop performance can be counter-intuitive and in general hard to predict. Instead, we propose to relax only the *minimization* of the objective function after $M$ time steps, but not the variables themselves. This relaxation of optimality reduces computation time by searching for an $N$-step feasible solution that is $M$-step optimal. While the theoretical worst-case runtime of this approach is the same as solving the original hybrid MPC problem, in practice the computational performance can be significantly improved if feasible trajectories can be found early in the branch-and-bound procedure. Moreover, the results on the closed-loop performance are predictable, allowing one to intuitively trade off computation time vs. closed-loop performance.

The second heuristic discussed in this paper revolves around the Cut function in the branch-and-bound algorithm, which is used to decide whether a subtree should be explored further or cut from the tree. A pre-processing procedure is proposed, which runs off-line and computes integer combinations that are infeasible or suboptimal, and hence will not be chosen by the optimizer. Using this information, a customized Cut function is proposed, which cuts a subtree whenever it encounters binary combinations that cannot be part of an optimal solution. For some problems, and in contrast