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A dynamic programming based approach for explicit model predictive control of hybrid systems

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ABSTRACT

This work presents an algorithm for explicit model predictive control of hybrid systems based on recent developments in constrained dynamic programming and multi-parametric programming. By using the proposed approach, suitable for problems with linear cost function, the original model predictive control formulation is disassembled into a set of smaller problems, which can be efficiently solved using multi-parametric mixed-integer programming algorithms. It is also shown how the methodology is applied in the context of explicit robust model predictive control of hybrid systems, where model uncertainty is taken into account. The proposed developments are demonstrated through a numerical example where the methodology is applied to the optimal control of a piece-wise affine system with linear cost function.

1. Introduction

The potential of using multi-parametric programming in the context of model predictive control has been widely documented in the literature (Bemporad et al., 2002b; Pistikopoulos et al., 2002, 2007). The idea is motivated by the fact that, in closed-loop model predictive control implementations, an optimisation problem needs to be solved whenever a sample of the system state is made available. For systems with fast sampling rates, the computational time required to solve the optimisation problem may become prohibitive, therefore rendering the use of model predictive control impractical.

By formulating the optimisation problem involved in model predictive control as a multi-parametric programming problem, with the state of the system being the vector of parameters (Bemporad et al., 2002b), it is possible to shift the computational effort involved in online optimisation to an offline step in which the optimal solutions for every possible realisation of the state vector are pre-computed as explicit functions. The regions in the state-space where these explicit functions are valid are referred to as critical regions. This method of designing controllers is called explicit model predictive control (Pistikopoulos et al., 2007).

The use of an explicit model predictive controller as a control device consists of evaluating the state of the system, at every sampling instance, and looking-up the corresponding optimal control input in the pre-computed map of critical regions. This operation is usually significantly faster than repeatedly solving optimisation problems, and therefore the method may be used for systems with faster sampling rates. According to a recent survey paper (Alessio and Bemporad, 2009), explicit model predictive controllers are suitable for applications with sampling rates as fast as 50 ms.

Currently available software tools for explicit model predictive control (ParOS, 2004; Kvasnica et al., 2004) may be used to design controllers with 1–2 input variables and 5–10 parameters (Alessio and Bemporad, 2009). However, one potential limitation is the rapid increase in the computational burden involved in solving the offline problem when the prediction horizon increases (Bemporad et al., 2002b).

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The combination of multi-parametric programming and dynamic programming (Bellman, 1957) has been reported as a method suitable for reducing the complexity of the optimisation problem involved in multi-stage decision processes, such as explicit model predictive control (Borrelli et al., 2005; Faísca et al., 2008; Kouramas et al., 2013). By using this method, the original problem is disassembled into a set of smaller sub-problems, with lower dimensionality, which are sequentially solved in a recursive manner. Recently, this approach has been extended to the problem of constrained dynamic programming of mixed-integer linear problems (Rivotti and Pistikopoulos, 2014).

Hybrid systems, i.e. systems involving continuous and discrete elements, find relevance in most processes of practical interest (Pantelides et al., 1999; Branicky et al., 1998). These often involve logical components and propositional logic statements that may be equivalently expressed through sets of linear constraints (Cavalier et al., 1990; Raman and Grossmann, 1991; Bemporad and Morari, 1999a). This property is explored by the mixed logical dynamical framework (Bemporad and Morari, 1999a) that provides a systematic method of converting logical propositions into a mixed-integer linear formulation (Bemporad et al., 2002b).

Due to this importance, including integer decision variables in an explicit model predictive control framework has been often identified as an important research direction (Morari and Lee, 1999; Pistikopoulos, 2009). However, the modelling of hybrid systems results in models with integer variables (Raman and Grossmann, 1992; Williams, 1999), and therefore in the need to use computationally complex multi-parametric mixed-integer programming algorithms to design the controllers. The problem of hybrid explicit model predictive control with a linear cost function has been addressed by Bemporad and Morari (1999a) and Baotic et al. (2006). Sakizlis et al. (2002) presented a method based on a mixed-integer quadratic programming algorithm (Dua et al., 2002) that handles quadratic cost functions.

In this publication, recent developments in constrained dynamic programming for mixed-integer linear systems (Rivotti and Pistikopoulos, 2014) are used as the foundations for a novel algorithm for hybrid explicit model predictive control with linear cost function. Additionally, it is shown how a closed-loop robust control policy may be obtained by considering modelling uncertainty within this framework.

The remaining sections are organised as follows. Fundamental concepts related to the modelling of hybrid systems are introduced in Section 2.1, with particular emphasis on piece-wise affine systems. The current literature and state of the art in hybrid explicit model predictive control is presented in Section 2.2. In Section 2.3, the main developments proposed are explained in detail and summarised in the form of an algorithm for hybrid explicit model predictive control with linear cost function. Different types of modelling uncertainty are introduced in Section 3.1 and Section 3.2 shows how the proposed algorithm is applied to the problem of robust explicit model predictive control of hybrid systems. Two numeric examples are presented in Section 4 to demonstrate the proposed developments. Concluding remarks and ongoing research topics are summarised in Section 5.

2. Explicit model predictive control for hybrid systems

2.1. Modelling of hybrid systems

Hybrid systems are an important topic in optimisation and control theory, and several classes of models are of a hybrid nature. In this section, the discussion is focused on piece-wise affine systems, a particular class of hybrid systems.

Piece-wise affine systems (Sontag, 1981; van Bokhoven, 1981; Johansson and Rantzer, 1998; Bemporad et al., 2000; Liberzon, 2003) are an important modelling tool and may be used, for example, to describe systems with nonlinear dynamics (Sontag, 1981).

A piece-wise affine state-space model is defined as shown in (1).

$$\begin{aligned} x_{k+1} &= A_i x_k + B_i u_k + c_i, \quad \forall (x_k, u_k) \in \mathcal{P}_i \\ y_k &= C_i x_k + D_i u_k \end{aligned} \tag{1}$$

In (1), $\mathcal{P}_i$ is a polyhedral partition of the space defined by the state and input of the system, and given by (2). The union of the polyhedra, $\mathcal{P} = \cup_{i=1,\ldots,p} \mathcal{P}_i$, is assumed to contain the origin.

$$P_i = \{(x_k, u_k) \mid F_i x_k + G_i u_k \le b_i, \quad i = 1, \ldots, p\} \tag{2}$$

Different values of the state-space matrices, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, $c_i \in \mathbb{R}^n$, $C_i \in \mathbb{R}^{n_y \times n}$, and $D_i \in \mathbb{R}^{n_y \times m}$ are triggered by the state of the system, $x_k \in \mathbb{R}^n$, and input of the system, $u_k \in \mathbb{R}^m$, reaching the corresponding partitions, P_i. In (2), F_i, G_i, and b_i, are constant matrices of appropriate dimensions.

One important property commonly required for the study of piece-wise affine systems is for (1) to be a well-posed system, in the sense of Definition 1.

Definition 1. Well-posed piece-wise affine system (Bemporad et al., 2000).

A piece-wise affine system, such as (1), is considered well-posed in $\mathcal{P}$ if, for all pairs $(x_k, u_k) \in \mathcal{P}$, the pair (x_{k+1}, u_{k+1}) exists and is uniquely determined.

As remarked in Bemporad et al. (2000), for (1) to be well-posed it is necessary for the definition of the partitions (2) to include both strict and non-strict inequalities.

For numerical applications involving optimisation problems, such as model predictive control, it becomes necessary to reformulate problems of a hybrid nature, such as (1), in a form suitable for the use of optimisation methods.

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