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Integrated planning and scheduling under production uncertainties: Bi-level model formulation and hybrid solution method

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ABSTRACT

We propose a novel method for integrating planning and scheduling problems under production uncertainties. The integrated problem is formulated into a bi-level program. The planning problem is solved in the upper level, while the scheduling problems in the planning periods are solved under uncertainties in the lower level. The planning and scheduling problems are linked via service level constraints. To solve the integrated problem, a hybrid method is developed, which iterates between a mixed-integer linear programming solver for the planning problem and an agent-based reactive scheduling method. If the service level constraints are not met, a cutting plane constraint is generated by the agent-based scheduling method and appended to the planning problem which is solved to determine new production quantities. The hybrid method returns an optimality gap for validating the solution quality. The proposed method is demonstrated by two complicated problems which are solved efficiently with small gaps less than 1%.

1. Introduction

Planning and scheduling are two core decision layers in a manufacturing organization (Wassick et al., 2012). Owing to different objectives and time scales, planning and scheduling problems are often solved separately in a sequential way. For example, a planning problem can be solved to determine weekly production quantities according to the customer orders. The output of the planning problem, the weekly production quantities, are then passed to the scheduling problem and a production schedule in each week is determined, such as assigning an operational task to a capable unit and sequencing the tasks in every unit. The sequential nature of the traditional approach prevents the planning model from using detailed production schedule information as the scheduling problem is not solved in the planning phase. Instead, the planning problem is solved based on aggregate information which is commonly a rough approximation of the real production data. For example, a planning model often includes a parameter to denote the time required to process a product but the actual processing time, due to product transition policies, is highly dependent on the production schedule. Though widely applied for its simplicity, the sequential methods often result in a suboptimal solution for the entire production process, or even an infeasible production plan that cannot be fulfilled by the scheduling procedure (Shen, Wang, & Hao, 2006; Tan & Khoshnevis, 2000).

To overcome the drawbacks of sequential methods, a great variety of integrated methods have been developed (Birewar & Grossmann, 1990; Maravelias & Sung, 2009; Phanden, Jain, & Verma, 2011; Shao, Li, Gao, & Zhang, 2009), which aim to improve the overall performance of the entire planning process by collaboratively solving the production planning problem and the scheduling problem. A major category of the integrated methods is the simultaneous methods, which solve a monolithic model formulated by combining all constraints of the planning model and the scheduling model. The computational difficulty arising from the formulated complex model can be addressed by a decomposition method (Li & Ierapetritou, 2009). The simultaneous methods can theoretically obtain the global optimal solution for the entire process. However, they encounter some practical difficulties due to the computational challenges (Shobrys & White, 2002). Implementation of the simultaneous method may require dismantling and reorganizing the existing production hierarchy in a company (Pinedo, 2009).

Other types of integrated methods, based on bi-level programming, have been proposed in the literature (Ryu, Dua, & Pistikopoulos, 2004). The planning problem is the upper level problem while the scheduling problem is the lower level problem, which is consistent with the existing production hierarchy. A bi-level optimization problem is closely related with a Stackelberg game which is played between a leader and multiple followers (Colson, Marcotte, & Savard, 2007). The planning problem acts as the leader,

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Nomenclature

Index
i processing unit (unit agent)
j, j' job (job agent)
m sample in the Monte Carlo method
n iteration in hybrid method
p product
r data set for the Monte Carlo method
s element in vector
t planning period

Parameter
A_{pt} unit processing time (aggregate information) for product p in period t
CAP_t total processing time (aggregate information) of period t
C_p^H unit hold cost for product p
C_p^S unit setup cost for product p
D_{pt} order demand of product p at the end of period t
E_s s-th column of identity matrix
H_t length of period t
M_{pt} upper bound of w_{pt}
N_{MC} number of sampling points in the Monte Carlo method
N_p number of products
N_R number of data sets
P_{MC} probability with which the results by the Monte Carlo method satisfy the probability constraints
P_t threshold of probabilistic constraint in period t in general formulation of integrated problem
PT_{ij} nominal processing time of task for job j processed in unit i
S_t^L threshold value of service level in period t
SUT_i nominal startup time of unit i
$TT_{ij'j}$ nominal transition time from job j' to job j in unit i

Variable
α_{tn} coefficient vector of linear bounding function in period t for iteration n
β_{pt} equal to 1 if product p is manufactured in period t
$cost$ total cost
ct_j completion time of job agent JA_j
δ_{tn} interception of linear bounding function in period t for iteration n
dt time step in agent-based simulation
inv_{pt} inventory level of product p at the end of period t
ms_t makespan of scheduling problem in period t
pm_t mean of pmc_t
pmc_t sample mean of η_t evaluated by the Monte Carlo method
$pmc_t^{(r)}$ sample mean of η_t evaluated by the Monte Carlo method using data set r
pv_t standard deviation of pmc_t
rpt_{ij} random disruption in PT_{ij}
$rsut_i$ random disruption in SUT_i
rt_i remaining time of unit agent UA_i to complete the being processed task
$rtt_{ij'j}$ random disruption in $TT_{ij'j}$
sn_j index to current task of job agent JA_j
st current time in agent-based simulation
θ_t vector of uncertain parameters in period t
w_{pt} production quantity of product p in period t
x^P vector of planning variables linked to scheduling problems
x_t^S vector of decision variables of scheduling problem in period t
y^P vector of planning variables other than x^P

Set
Set_Waiting_Job index set of jobs whose current tasks wait for processing
Set_Ready_Unit index set of idle units ready to process a task
Set_Unit_Job set of (i, j) such that current task of JA_j can be processed in UA_i

Function
η_t function in probabilistic constraints in general formulation of integrated problem
f objective function of upper level problem in general formulation of integrated problem
φ_t objective function of lower level problem in period t in general formulation of integrated problem
Φ inverse cumulative distribution function of the standard normal random variable
ψ_t scheduling model in period t in general formulation of integrated problem
g planning model in general formulation of integrated problem
Pr_{θ_t} probability under uncertain parameters in θ_t

while the scheduling problem that consists of subproblems over the planning period acts as the follower. The scheduling problems in each time period are solved for their own objectives. However, their solutions (called responses in the game theory) can typically be governed by the planning problem, which is solved to optimize the entire production process. These bi-level programming methods can be regarded as a compromise between the simultaneous methods and the sequential methods. Similar to the simultaneous methods, they optimize the entire production process as an integrated problem. Moreover, these methods grant a degree of autonomy to the scheduling problems to have their own objectives like the sequential methods.

The degree of autonomy is crucial for the scheduling problems in order to deal with production uncertainties. When disruptions occur, the process is often rescheduled accordingly. The reactive scheduling decisions are commonly made autonomously according to a local objective because these decisions are made in real-time to adjust to the circumstances as they occur. Solving a complex monolithic problem to optimize a single objective under disruptions specific to scheduling in each time period is often impractical. Compared to the simultaneous methods, different objectives stemming from reactive scheduling are readily accommodated by the bi-level programming methods.

Though attractive because the model formulation aligns with the exiting organizational structure in production companies, application of the bi-level programming methods encounters some obstacles. The first challenge is computational complexity. A bi-level program is intrinsically difficult to solve. Even the simplest instance, the linear bi-level program where all problems in both upper level and lower level are linear with only continuous decision variables, has been shown to be NP-hard (Hansen, Jaumard, & Savard, 1992). Given that either a planning problem or a scheduling problem can be NP-hard (Ullman, 1975), the integrated planning and scheduling problem formulated as a bi-level program is mathematically intractable in general.