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Unsteady conjugated heat transfer in thick walled pipes involving two-dimensional wall and axial fluid conduction with uniform heat flux boundary condition

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ABSTRACT

Transient conjugated heat transfer in thick walled pipes for thermally developing laminar flow is investigated involving two-dimensional wall and axial fluid conduction. The problem is solved numerically by a finite-difference method for hydrodynamically developed flow in a two-regional pipe, initially isothermal in which the upstream region is insulated and the downstream region is subjected to a suddenly applied uniform heat flux. A parametric study is done to analyze the effects of four defining parameters namely, wall thickness ratio, wall-to-fluid thermal conductivity ratio, wall-to-fluid thermal diffusivity ratio and the Peclet number. The results are given by non-dimensional interfacial heat flux values, and it is observed that, heat transfer characteristics are strongly dependent on the parameter values.

1. Introduction

Analysis of conjugated heat transfer in transient regime is important during start up, shutoff or any change in the operating conditions. This problem may be faced in regenerative and recuperative heat exchangers, in cooling of gas turbine blades, in nuclear reactors, aircraft engines and spacecrafts, and is more likely to be analyzed in pipes or in flow sections which can be modeled as a pipe or channel.

Transient heat transfer for laminar pipe or channel flow was analyzed by many investigators and in some of them the pipe wall is considered extremely thin. In this case the wall conduction may be ignored and the condition at the outer wall surface can be assumed to prevail along the inner surface. However, in thick walled pipes the conditions at the wall–fluid interface are not known a priory and the energy equations must be solved simultaneously by assuming continuity in temperatures and in heat fluxes at the interface. When Peclet number of the flow is low, the axial fluid conduction may be comparable to convection and can not be ignored. Diffusion of heat backward through the upstream region, results preheating of the fluid before the beginning of the heating section. Therefore such problems are usually analyzed in two-regional pipes. A brief literature survey on steady conjugated problems and on the effect of axial fluid conduction is given in [1,2]. A more recent survey on the subject may also be found in the paper of Weigand and Gassner [3]. They studied a conjugate extended Graetz problem analytically for pipes with a step change in constant outside wall temperature in a finite region.

Unsteady conjugated problems for laminar flow considering one or two-dimensional wall conduction and fluid axial conduction were also studied by many investigators under various boundary conditions. Schneider [4] solved the problem for parallel plates and Vick et al. [5] for pipes with uniform flow and convection from the outer surface by analytical methods. Campo and Auguste [6] worked on a problem with parabolic velocity profile and both with convective and radiative boundary conditions. Numerical methods are used for solving the problem in pipes, heated in finite length, with a step change in heat flux, by Lin and Kuo [7] and in temperature, by Yan et al. [8]. With variable inlet fluid conditions in parallel plates, the problem is investigated by Travelho and Santos [9], with uniform flow, and by Olek et al. [10], with parabolic flow. Yapıcı and Albayrak [11] solved a problem with non-uniform heat fluxes and Yin and Bau [12] with and without axial fluid conduction.

Recently numerical methods were used in investigations considering two-dimensional wall and axial fluid conduction. Schutte et al. [13] solved the problem, for combined development region, Lee and Yan [14], Bilir [15] and Zueco et al. [16], with step change in wall temperature, Yan [17] and Bilir and Ateş [18], with convective boundary conditions and Li and Kakaç [19], with step and sinusoidal change in wall heat flux, Luna et al. [20], for power-law fluids with step change in wall heat flux.

2. Problem formulation

The schematics of the problem and the coordinate system are shown in Fig. 1. The flow pipe is two-regional and infinite in

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Nomenclature

a	constant of discretization equation (Eqs. (6a)–(6j))
c_p	specific heat at constant pressure
d	thickness of the pipe wall
Fo	Fourier number
Gz	Graetz number
k	thermal conductivity
Nu	Nusselt number
Pe	Peclet number
q	heat flux
r	radial coordinate
t	time
T	temperature
T_0	initial temperature of the system
u	axial velocity
x	axial coordinate
Greek symbols	
α	thermal diffusivity
δr	radial position difference
δx	axial position difference
Δr	radial step size
Δt	time step increment
Δx	axial step size
ε	relative error
ρ	density
Subscripts	
b	bulk
f	fluid
i	inner wall
i, j	at nodal point i, j
m	mean
o	outer wall
w	wall
wf	ratio of wall to fluid
Superscripts	
$'$	dimensionless quantity
0	at previous time step

both sides. At the far upstream, the fluid temperature is T_0 and uniform. The upstream region of the pipe wall is externally insulated; the flow is laminar and hydrodynamically developed at the beginning of the downstream region. Initially the whole system is isothermal at temperature T_0, and at time $t = 0$ a constant and uniform heat flux q_{wo} is suddenly applied on the external surface of the downstream side of the pipe. Physical properties of the fluid are assumed to be constant and the viscous dissipation is neglected.

The above-described problem may be formulated in non-dimensional form as follows. In the wall side, the differential equation is

$$\frac{1}{\alpha_{wf}}\frac{\partial T'_w}{\partial t'} = \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial T'_w}{\partial r'}\right) + \frac{1}{Pe^2}\frac{\partial^2 T'_w}{\partial x'^2}. \quad (1a)$$

The initial and boundary conditions are

$$\text{at } t' = 0 \quad T'_w = 0; \quad (1b)$$

$$\text{at } x' = -\infty \quad T'_w = 0; \quad (1c)$$

$$\text{at } x' = +\infty \quad \frac{\partial T'_w}{\partial x'} = 4(1 + d'); \quad (1d)$$

$$\text{at } r' = 1 + d' \quad \text{for } x' < 0 \quad \frac{\partial T'_w}{\partial r'} = 0; \quad (1e)$$

$$\text{at } r' = 1 + d' \quad \text{for } x' \geqslant 0 \quad \frac{\partial T'_w}{\partial r'} = \frac{1}{k_{wf}}; \quad (1f)$$

$$\text{at } r' = 1 \quad T'_w = T'_f \quad \text{and} \quad \frac{\partial T'_w}{\partial r'} = \frac{1}{k_{wf}}\frac{\partial T'_f}{\partial r'}. \quad (1g, h)$$

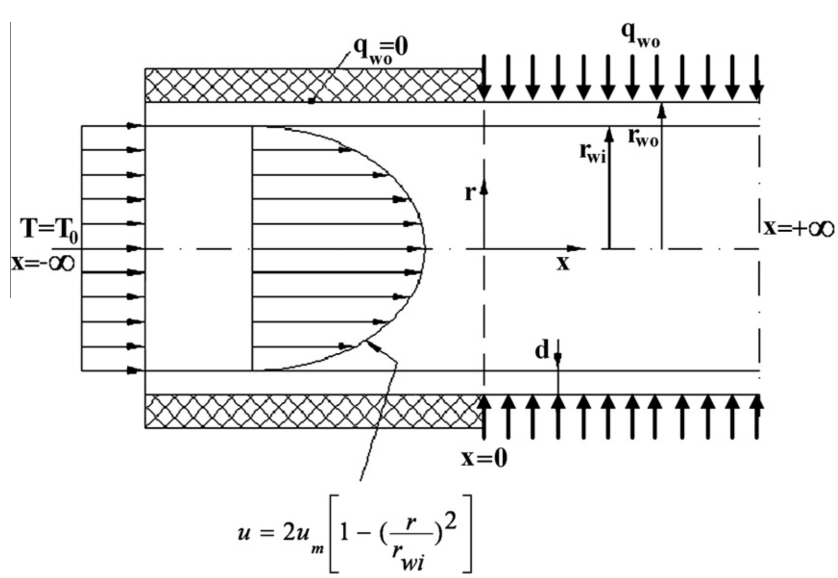

Fig. 1. Schematics of the problem and the coordinate system.