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Onset of buoyancy-driven motion with laminar forced convection flows in a horizontal porous channel

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ABSTRACT

Buoyancy-driven motion in the laminar forced convection flow has been investigated in a horizontal porous channel. The stability equations including the inertia and the dispersion effects have been solved analytically under the linear theory and also the principle of exchanges of stabilities. The resulting critical position x_c for the onset of convection becomes larger with increasing the Reynolds number, Re_K considering the permeability. This means that the more inertia and dispersion make the system more stable. The ionic mass transfer experiments using the limiting current technique have been conducted to obtain the critical position for the manifest convection, i.e. the undershoot distance x_u with comparison of the forced convective mass flux with the mixed-convective one. Based on the experimental results, the inertia and dispersion effects are explained reasonably for the stabilization of the present system.

1. Introduction

The convective motion driven by the buoyancy forces appears in the fluid-saturated porous layer subjected to heating from below. This is well known as the Horton–Rogers–Lapwood problem [1,2]. The convection in porous media plays an important role in science and engineering applications, such as mantle recurrence, oil recovery, storage of grain and nuclear reactor safety. An interesting extension of the Horton–Rogers–Lapwood problem is the mixed convection under the weak through flow [3]. Many studies on the transient behaviors the mixed convection in the porous layer have been carried out after Prats [4] analyzed the time-dependent oscillating motion. Combarnous and his colleagues [5,6] showed that the phase transition of the time-dependent transverse rolls to the steady longitudinal vortices appears while the rate of through flow increases. Dufour and Neel [7] suggested the criteria on the absolute and convective instabilities using the weakly-nonlinear analysis. Recently, Chung et al. [8] suggested the reasonable stability criteria for the fluid-saturated porous media under the forced convection using both linear and weakly nonlinear analyses, and also the direct numerical simulation. According to the survey of previous analyses, it seems that the longitudinal vortices dominate the transverse rolls for the specific region where an inertial effect is dominant.

These works are mainly focused on the systems with the fully-developed base fields. For the various systems experiencing the developing nonlinear base fields, Choi and his colleagues [9–12] have analyzed the onset of the vortex instabilities successfully by employing their propagation theory. Lee et al. [11] examined the laminar vortex instability on the natural convection flow over the inclined surface embedded the porous layer, where the limiting current method in an electrochemical system has been done for promising the trend line of their stability conditions consistently. For a forced convection flow through the porous media, Chung et al. [12] analyzed the onset of the convective instability including the inertia and the dispersion effects. They showed that these effects make the system stable.

The experimental detection on the natural convection for a high Rayleigh number is very difficult due to the side effects and the difficulties in the control of boundary conditions. Furthermore, the observation of the convective motion in the porous media is hardly detected due to the structural complexity. To overcome the above mentioned problems, electrochemical systems [13–15] under the limiting current condition [16] have been used in the natural convection fields especially for the very large Rayleigh number situations. Lee et al. [11] showed that this method can be extended to the system of the fluid-saturated porous media. In the porous media systems, the dispersion effect caused by the variation of

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Nomenclature

a	dimensionless wave number
a^*	modified wave number, $a\sqrt{(1+\gamma_L)x}$
C	concentration (mol/m^3)
c_F	dimensionless form drag constant
c_0	dimensionless base concentration, $(C_0 - C_i)/\Delta C$
c	dimensionless concentration disturbance, $RaC_1/\Delta C$
$\mathbf{D}$	the dispersion tensor (m^2/s)
d	particle diameter (m)
D_e	effective diffusivity (m^2/s)
Da	Darcy number, K/d^2
g	gravitational acceleration (m/s^2)
H	fluid layer depth (m)
h_L	average mass transfer coefficient (m/s)
K	permeability (m^2)
P	pressure (Pa)
Pe	Péclet number, U_0H/D_e
Pe_d	Péclet number based on particle diameter, U_0d/D_e
Pe_K	Péclet number based on permeability, $U_0c_F\sqrt{K}/D_e$
Ra	Rayleigh number, $\rho g\beta\Delta CH^3/(D_e\mu)$
Ra_D	Darcy–Rayleigh number, $DaRa$
Ra_D^*	modified Darcy–Rayleigh number, $Ra_Dx^{1/2}/(1+Re_K)\sqrt{1+\gamma_L}$
Re_d	Reynolds number based on particle diameter, $\rho U_dd/\mu$
Re_K	Reynolds number based on permeability, $\rho U_0c_F\sqrt{K}/\mu$
Sc	Schmidt number, $\mu/(\rho D_e)$
Sh	Sherwood number, h_LH/D_e
t	time (s)
(U,V,W)	velocities in Cartesian coordinates (m/s)
(u,v,w)	dimensionless velocity disturbances in Cartesian coordinates
(X,Y,Z)	Cartesian coordinates (m)
(x,y,z)	dimensionless Cartesian coordinates

Greek symbols

β	concentration expansion coefficient (m^3/mol)
γ	ratio of dispersion coefficient to effective diffusivity, $\mathbf{D}/D_e - 1$
Δ_C	concentration boundary-layer thickness (m)
ε	porosity
ζ	dimensionless similarity variable, $z/\sqrt{(1+\gamma_L)x}$
μ	viscosity (Pa s)
ρ	density (kg/m^3)
τ	dimensionless time, D_et/H^2

Subscripts

0	basic quantities
1	perturbed quantities
b	bulk condition
c	critical conditions
e	effective properties
i	inlet state
L	longitudinal direction
l	lower plate
T	transverse direction
u	upper plate

Superscript

*	quantities in (x,ζ)-domain

porosity, the thermal deviation between solid and fluid, and the fluid velocity [17–20] plays an important role in the determination of stability criteria. But, the stabilization from the buoyancy forces due to the dispersion effects is not examined yet for the convective flows in a horizontal porous channel.

In the present study, we will analyze the onset of a buoyancy-driven instability for the laminar convection flow through the porous layers. The stability condition from the linear stability analysis will be compared with the electrochemical mass transfer experiment. Based on the comparison between the theoretical and experimental results, the inertia and the dispersion effects on the onset of buoyancy-driven secondary motion will be quantified.

2. Theoretical analysis

2.1. Governing equations

The mass transfer system considered here is a fluid-saturated porous layer with uniform superficial velocity U_0 and uniform concentration in C_i the fully-developed laminar flow (see Fig. 1). The porous layer is confined between two horizontal plates of depth H. The lower and upper plates are kept at constant concentration C_l and C_u, respectively. For a small distance in the streamwise X-direction, the nonlinear concentration profile develops gradually. For $X > 0$, the laminar concentration boundary-layer thickness Δ_C increases with increasing X, and the buoyancy-driven secondary flow will set in at a certain distance. For the isotropic porous media, the governing momentum equation is expressed using the Forchheimer's equation, Boussinesq approximation and dispersion model:

$$\frac{\mu}{K}\mathbf{U} + \frac{c_F\rho}{\sqrt{K}}|\mathbf{U}|\mathbf{U} = -\nabla P - \mathbf{k}\rho\beta gC, \tag{1}$$

where $\mathbf{U}$ is the superficial velocity vector and $\mathbf{k}$ is the unit vector of positive Z-direction. And the general governing equations for the continuity and mass balances are as follows:

$$\nabla\cdot\mathbf{U} = 0, \tag{2}$$

$$\frac{\partial C}{\partial t} + \mathbf{U}\cdot\nabla C = \nabla\cdot(\mathbf{D}\nabla C), \tag{3}$$

where t and $\mathbf{D}$ denote the time and the dispersion tensor, respectively. Hsu and Cheng [17] suggested that $\mathbf{D} = D_e + \mathbf{A}Pe_d^n$, where D_e is the effective diffusivity, and $\mathbf{A}$ is the constant 2nd-order tensor which is diagonal. They assumed $n = 1$ for high Re_d and $n = 2$ for low Re_d. Here, $Pe_d(= U_0d/D_e)$ and $Re_d(= \rho U_dd/\mu)$ is the Péclet number and the Reynolds number based on the particle diameter d, respectively. $Pe_K\left(= U_0c_F\sqrt{K}/D_e\right)$ and $Re_K\left(= \rho U_0c_F\sqrt{K}/\mu\right)$ which use $\sqrt{K}$ as a length scale is more general parameters for the non-spherical porous matrices.

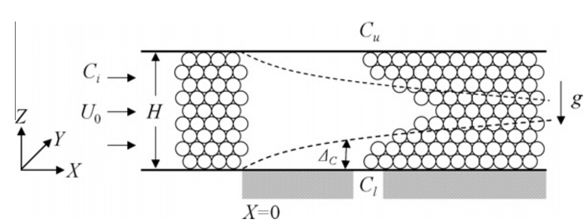

Fig. 1. Schematic diagram of the system considered here.