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Recursive constrained state estimation using modified extended Kalman filter

J. Prakash[a], Biao Huang[b,*], Sirish L. Shah[b]

[a] Department of Instrumentation Engineering, Madras Institute of Technology Campus, Anna University, Chennai 600044, India
[b] Department of Chemical and Materials Engineering, University of Alberta, Edmonton T6G 2G6, Canada

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ABSTRACT

The extended Kalman filter (EKF) remains the most preferred state estimator for solving both unconstrained and constrained state estimation problems in the field of Chemical Engineering. Given, the wide spread use of EKF, we have proposed a novel optimization free recursive formulation of the EKF, to handle elegantly bounds on the estimated state variables of a stochastic non-linear dynamic system. It is well known that in the EKF, the prior and posterior distributions are approximated to be a multivariate normal distribution. In the presence of bounds imposed on the state variables, the accuracy of the first two moments of the initial state distribution and prior distribution namely the means and covariance matrices, plays a significant role in the extended Kalman filter performance. Hence, in this paper, we propose two novel schemes to modify the prior and posterior distributions of the EKF in order to satisfy the bound constraints. In addition, the initial state distribution is also suitably modified in order to satisfy the bound constraints. The efficacy of the proposed state estimation schemes using the EKF is validated on two benchmark problems reported in the literature namely a simulated gas-phase reactor and an isothermal batch reactor involving constraints on estimated state variables. Extensive simulation studies show the effectiveness of the proposed optimization free recursive constrained state estimation schemes using extended Kalman filter.

1. Introduction

Nonlinear state estimation is an active research area and significant amount of work has been carried out in the Chemical Engineering discipline over the last four decades to develop methods for the estimation of state variables and parameters in nonlinear dynamic systems (Dochain, 1998; Patwardhan, Narasimhan, Jagadeesan, Gopaluni, & Shah, 2012; Rawlings & Bakshi, 2006).

The methods for the estimation of state variables and parameters in stochastic nonlinear dynamic systems can be broadly categorized into unconstrained state estimation schemes and constrained state estimation schemes. The most common recursive unconstrained state estimation schemes are extended Kalman filter (EKF) (Gelb, 1974), unscented Kalman filter (UKF) (Julier & Uhlmann, 2004), ensemble Kalman filter (EnKF) (Evenson, 2003) and particle filter (Arulampalam, Maskell, Gordon, & Clapp, 2002). Several variants of EKF, UKF and PF have been also proposed in the literature (Simon, 2006). However, the above-mentioned unconstrained methods do not deal with bounds and constraints imposed on state variables and parameters in a systematic way (Abrol & Edgar, 2011; Haseltine & Rawlings, 2005; Rao, Rawlings, & Mayne, 2003; Rawlings & Bakshi, 2006; Vachhani, Rengaswamy, Gangwal, & Narasimhan, 2004; Vachhani, Narasimhan, & Rengaswamy, 2006).

In order to deal with constraints, which arise in most Chemical Engineering processes, plethora of methods have been proposed, namely, moving horizon estimation (MHE) (Rao et al., 2003), recursive non-linear dynamic data reconciliation (RNDDR) (Vachhani et al., 2004), unscented recursive nonlinear dynamic data reconciliation (URNDDR) (Vachhani et al., 2006), constrained derivative-free Kalman filters (Kandepu, Foss, & Imsland, 2008; Kolås, Foss, & Schei, 2009; Prakash, Patwardhan, & Shah, 2010; Teixeira, Tôrres, Aguirre, & Bernstein, 2010), constrained particle filters (Prakash, Patwardhan, & Shah, 2011; Shao, Huang, & Lee, 2010; Stano, Lendek, & Babuška, 2013). MHE while being considered as a very popular constrained state estimation technique does not follow a predictor-corrector approach whereas, RNDDR, URNDDR, constrained derivative-free Kalman filters and particle filters follow a predictor-corrector approach. For the computation of the arrival

* Corresponding author. Tel.: +1 780 492 9016.
E-mail addresses: bhuang@ualberta.ca, biao.huang@ualberta.ca (B. Huang).

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cost in MHE, constrained recursive state estimators have been recently used (López-Negrete, Patwardhan, & Biegler, 2011). In all the above methods, nonlinearity and constraints on state variables and parameters have been handled elegantly by posing the state and parameter estimation problem in an optimization framework (Patwardhan et al., 2012).

Recursive nonlinear dynamic data reconciliation has been recently developed (Vachhani et al., 2004), which combines computational advantages of recursive estimation while handling constraints on the state variables. In the correction step of RNDDR, an optimization problem is solved to update the predicted state estimates using measurements. The computation of one-step ahead predicted state estimate ($\hat{\mathbf{x}}(k|k-1)$), predicted error covariance matrix ($P(k|k-1)$) and updated error covariance matrix $P(k|k)$ in RNDDR formulation are identical to that of the extended Kalman filter. RNDDR also requires computing derivatives of nonlinear state transition function and nonlinear measurement model, and is accurate to the first order. Hence, RNDDR can be viewed as one form of a constrained extended Kalman filter. Vachhani et al. (2006) later proposed unscented recursive nonlinear dynamic data reconciliation (URNDDR) to estimate the state variables and parameters of the nonlinear system by combining the advantages of the unscented Kalman filter (UKF) and RNDDR. Modified versions of RNDDR and URNDDR have been recently proposed (Kadu, Bhushan, Gudi, & Roy, 2010; Rengaswamy, Narasimhan, & Kuppuraj, 2011).

It should be noted that the extended Kalman filter still remains as the most preferred state estimator for solving constrained state estimation problems in the field of chemical engineering (Dewasme, Goffaux, Hantson, & Vande Wouwer, 2013; Khodadadi & Jazayeri-Rad, 2011). In the EKF, to the best of our knowledge, the bounds have been handled either by clipping only the state estimate or by solving the on-line optimization problem as suggested in RNDDR. In both the approaches, the bounds and constraints have not been accounted while computing the covariance matrices of the errors in the predicted ($P(k|k-1)$) and updated state estimates ($P(k|k)$, innovation covariance matrix ($V(k)$), Kalman gain ($K(k)$) and one-step ahead predicted state estimate ($\hat{\mathbf{x}}(k|k-1)$).

It should be noted that the accuracy of the first two moments of the initial state distribution $p[\mathbf{x}(0)]$ and prior distribution $p[\mathbf{x}(k)|\mathbf{Y}^{k-1}]$, namely the means and covariance matrices, play a significant part in the extended Kalman filter performance (Schneider & Georgakis, 2013). We propose two novel schemes to approximate the prior and posterior distributions to be a truncated multivariate normal distribution (Robert, 1995; Kotecha & Djuric, 1999; Wilhelm & Manjunath, 2010) through (i) generation of samples $\{\hat{\mathbf{x}}_c^i(k|k-1)\}$ from the truncated multivariate normal distribution $N_T[\hat{\mathbf{x}}(k|k-1), P(k|k-1), x_L, x_H]$, computation of first two moments of the truncated prior distribution such as $\hat{\mathbf{x}}_c(k|k-1)$, $P_c(k|k-1)$, respectively, from the randomly drawn samples and use them to compute the innovation and Kalman gain (ii) generation of samples $\{\hat{\mathbf{x}}_c^i(k|k-1)\}$ from the multivariate normal distribution $N[\hat{\mathbf{x}}(k|k-1), P(k|k-1)]$, while those samples lying outside the bounds are clipped; then the first two moments of the truncated prior distribution are computed and being used in the computation of the innovation and Kalman gain. In addition, the first two moments of the initial state distribution $p[\mathbf{x}(0)]{\sim}N[\hat{\mathbf{x}}(0), P(0)]$ and posterior distribution are also modified in order to satisfy the bound imposed on the state variables. The efficacy of the proposed approaches is demonstrated using following benchmark problems in the literature:

- gas-phase reactor (Rawlings & Bakshi, 2006);
- isothermal batch reactor (Haseltine & Rawlings, 2005).

The organization of the paper is as follows. Section 2 discusses the recursive Bayesian state estimation and presents the unconstrained EKF formulation. The proposed constrained state estimation schemes based on EKF are presented in Section 3. The simulation results are presented in Section 4 followed by main conclusions drawn through the analysis of these results as discussed in Section 5.

2. Extended Kalman filter

Consider a nonlinear system represented by the following discrete nonlinear state space equations:

$$\mathbf{x}(k) = \left[\mathbf{x}(k-1) + \int_{k-1}^{k} \mathbf{F}[\mathbf{x}(\tau), \mathbf{u}(k-1)]\, d\tau\right] + \mathbf{w}(k) \quad (1)$$

and

$$\mathbf{y}(k) = \mathbf{H}[\mathbf{x}(k)] + \mathbf{v}(k) \quad (2)$$

$$\mathbf{x}_L \le \mathbf{x}(k) \le \mathbf{x}_H \quad (3)$$

In the above equations, $\mathbf{x}(k)$ is the system state vector ($\mathbf{x} \in R^n$), $u(k)$ is known input ($\mathbf{u} \in R^m$), $\mathbf{w}(k)$ is the process noise ($\mathbf{w} \in R^n$) with known distribution, $\mathbf{y}(k)$ is the measured variable ($\mathbf{y} \in R^r$) and $\mathbf{V}(k)$ is the measurement noise ($\mathbf{v}(k) \in R^r$) with known distribution. The index 'k' represents the sampling instant and the symbols **F** and **H** are namely the state transition function and measurement function, respectively, and is assumed to be known in this work. It is further assumed that the initial state of the system $\mathbf{x}(0)$ is a random vector with known probability distribution. It may be noted that the process noise and measurement noise have been assumed to influence the system dynamics and measurement map in an additive manner.

The objective of the recursive Bayesian state estimation problem is to find the mean and variance of a random variable $\mathbf{x}(k)$ using the conditional probability density function $p[\mathbf{x}(k)|\mathbf{Y}^{(k)}]$. $\mathbf{Y}^{(k)}$ denotes the set of all the available measurements, i.e. $\mathbf{Y}^{(k)} \triangleq \{\mathbf{y}(k), \mathbf{y}(k-1), \ldots\}$. The posterior density $p[\mathbf{x}(k)|\mathbf{Y}^k]$ is estimated in two steps namely the prediction step and the update step. In the prediction step, the posterior density $p[\mathbf{x}(k-1)|\mathbf{Y}^{k-1}]$ at the previous time step is propagated into the next time step through the transition density $\{p[\mathbf{x}(k)|\mathbf{x}(k-1)]\}$ as follows:

$$p[\mathbf{x}(k)|\mathbf{Y}^{k-1}] = \int p[\mathbf{x}(k)|\mathbf{x}(k-1)]p[\mathbf{x}(k-1)|\mathbf{Y}^{k-1}]\, d\mathbf{x}(k-1) \quad (3)$$

The update stage involves the application of Bayes' rule:

$$p[\mathbf{x}(k)|\mathbf{Y}^k] = \frac{p[\mathbf{y}(k)|\mathbf{x}(k)]}{p[\mathbf{y}(k)|\mathbf{Y}^{k-1}]} \times p[\mathbf{x}(k)|\mathbf{Y}^{k-1}] \quad (4)$$

It should be noted that the properties of the state transition equation (1) are accounted through the transition density function $p[\mathbf{x}(k)|\mathbf{x}(k-1)]$ while $p[\mathbf{y}(k)|\mathbf{x}(k)]$ accounts for the nonlinear measurement model. The prediction and update strategies provide an optimal solution to the state estimation problem and the analytical solution to the recursive propagation of the posterior density is difficult to obtain. However, when the process model is linear and process and measurement noise sequences are zero mean Gaussian white noise sequences, and in the absence of state constraints, the Kalman filter describe the optimal recursive solution to the sequential state estimation problem (Soderstorm, 2002).

The extended Kalman filter (EKF) is probably the most widely used nonlinear filter. We have assumed in this work that the initial state and the sequence $\{w(k)\}$ and $\{v(k)\}$ are white, Gaussian, and independent of each other.

$$E[w(k)] = 0 \quad (5)$$