Computers and Chemical Engineering 65 (2014) 102–111

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Optimal multi-scale capacity planning for power-intensive continuous processes under time-sensitive electricity prices and demand uncertainty. Part II: Enhanced hybrid bi-level decomposition

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ARTICLE INFO

Article history:
Received 24 May 2013
Received in revised form 11 February 2014
Accepted 14 February 2014
Available online 24 February 2014

Keywords:
Stochastic programming
Integer recourse
Decomposition
Demand-side management
Smart grid

ABSTRACT

We describe a hybrid bi-level decomposition scheme that addresses the challenge of solving a large-scale two-stage stochastic programming problem with mixed-integer recourse, which results from a multi-scale capacity planning problem as described in Part I of this paper series. The decomposition scheme combines bi-level decomposition with Benders decomposition, and relies on additional strengthening cuts from a Lagrangean-type relaxation and subset-type cuts from structure in the linking constraints between investment and operational variables. The application of the scheme with a parallel implementation to an industrial case study reduces the computational time by two orders of magnitude when compared with the time required for the solution of the full-space model.

1. Introduction

In Part I of the paper, we described a model for the integration of operational and strategic decision-making for continuous power-intensive processes under time-sensitive electricity prices and product demand uncertainty. The resulting formulation is a two-stage stochastic programming problem (Birge & Louveaux, 2011), whose deterministic equivalent is a large-scale MILP due to the integration of different time-scales, from hourly decisions on production levels and modes to investments decisions over a horizon of multiple years. Therefore, the problem is hard to solve and it already has nearly 1 million constraints, 2.4 million variables (of which 221,780 are binary) for the case of 60 scenarios, which result from modeling a ten year horizon with an aggregated time representation and three scenarios per season. At the same time, the problem has a structure that deserves special attention. Similar to other two-stage stochastic programming problems, the problem decomposes into individual operational subproblems once first-stage investment decisions are fixed. However, one major challenge is the large number of binary decision variables in the second stage that originate from detailed scheduling problems.

There are two main decomposition schemes that have been applied to two-stage stochastic programming problems, namely Lagrangean decomposition (Caroe & Schultz, 1999; Guignard & Kim, 1987; Guignard, 2003) and Benders decomposition also known as L-shaped method (Benders, 1962; Geoffrion, 1972; Van Slyke & Wets, 1969).

Lagrangean decomposition applied to stochastic programming problems is a special form of Lagrangean relaxation, which decomposes the original problem into subproblems by duplicating the first-stage investment variables and dualizing the so-called non-anticipativity constraints that enforce the same first-stage investment decisions across all scenarios. The multipliers of the dualized non-anticipativity constraints (the so-called complicating constraints) are iteratively updated with subgradient optimization or cutting planes. Lagrangean decomposition can also be applied for non-convex problems. However, the duality gap that arises can only be closed by using a branch-and-bound enumeration in the full variable space (Karuppiah & Grossmann, 2008).

Benders decomposition solves the original problem by evaluating the second-stage subproblems for different realizations of the complicating variables. The search in the space of the complicating variables is performed with a master problem that collects dual information from the subproblems, which describe the sensitivity of the second-stage decisions with respect to the first-stage decisions. Note that the collection of dual information relies on strong duality and becomes difficult if the problem is non-convex in the

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http://dx.doi.org/10.1016/j.compchemeng.2014.02.012
0098-1354/

second-stage variables. Recently, Li, Tomasgard, and Barton (2011) extend the idea of Benders decomposition to non-convex problems by replacing the original non-convex problem by a convex relaxation and applying Benders decomposition to the relaxation. Sundaramoorthy, Li, Evans, and Barton (2012) apply Li et al.'s algorithm to a two-stochastic programming problem that represents a capacity planning problem in the pharmaceutical industry.

Another decomposition scheme, which in contrast to Benders and Lagrangean decomposition does not rely on dual information, is the so-called bi-level decomposition algorithm (Iyer & Grossmann, 1998). Bi-level decomposition has been used in various applications, ranging from investment planning problems for utility plants (Iyer & Grossmann, 1998), oil fields (van den Heever & Grossmann, 1999), and supply chains (You, Grossmann, & Wassick, 2011) to planning and scheduling problems (Erdirik-Dogan & Grossmann, 2008). The algorithm is also based on the idea that some decision variables of the problem are complicating variables, e.g. investment decisions in strategic planning problems or assignment variables in planning and scheduling problems.

However, in contrast to Benders decomposition, the master problem is an aggregated problem (AP) that corresponds to a tailored relaxation of the original problem, typically obtained by relaxing 0/1 variables, relaxing some constraints and/or taking linear combinations of them. The aggregated problem (AP) yields an initial bound, normally much tighter than the one obtained from the initial Benders master problem. AP is solved alternately with the detailed problem (DP), in which the complicating variables are restricted. Primal cuts are inferred from DP and added back to AP, and the process iterates until the gap between the objective function values of the two problems is within a specified tolerance. Recently, Calfa, Agarwal, Grossmann, and Wassick (2013) as well as Terrazas-Moreno and Grossmann (2011) apply Lagrangean decomposition within bi-level decomposition to the aggregated problem (AP). While both authors can speed up the solution process significantly, their schemes lead to duality gaps due to the application of Lagrangean decomposition.

In this part of the paper, we focus on the development and application of a suitable decomposition strategy for the two-stage multi-scale stochastic programming problem that we described in Part I of the paper. We intend to combine the individual strengths of the afore mentioned decomposition schemes. First, the problem statement is reviewed in Section 2. The classical bi-level decomposition algorithm is introduced in Section 3 and applied to our problem. We derive subset-type cuts for the second-stage value function based on the solution of the detailed problem (DP). In Section 4, we explain how a Lagrangean-type relaxation of DP can be used to generate good initial bounds, and how Benders decomposition is used to further decompose the aggregated problem (AP). The complete enhanced hybrid bi-level decomposition algorithm is described in Section 5. Finally, in Section 6, we discuss the parallel implementation of our scheme within the GAMS grid computing environment (Bussieck, Ferris, & Meeraus, 2009), and show computational results that demonstrate the impact of our decomposition algorithm for the multi-scale capacity planning problem applied to the industrial case study from Part I of this paper.

2. Problem statement

We would like to solve a two-stage stochastic programming problem for the multi-scale capacity planning of a continuous power-intensive process under time-sensitive electricity prices and product demand uncertainty. The scenarios correspond to different demand realizations over the time horizon and the problem has complete recourse since variables for external product purchases with associated cost terms in the objective function are present. The problem we introduced in Part I of this paper can be summarized in the following way:

$$(P) \quad \min \sum_{t' \in T_{invest}} c_{t'}^T x_{t'} + \sum_{t \in T, s \in S} \tau_{t,s} d_{t,s}^T y_{t,s} \tag{1}$$

$$\text{s.t.} \sum_{t' \in T_{invest}} A_{0,t'} x_{t'} \quad \leq b_0 \tag{2}$$

$$\sum_{t' \in T_{invest}, t' \leq t} A_{1,t'} x_{t'} + B_1 y_{t,s} \leq b_1 \quad \forall t \in T, s \in S \tag{3}$$

$$y_{t,s} \in Y_{t,s} \quad \forall t \in T, s \in S \tag{4}$$

$$x_{t'} \in \{0,1\}^n \quad \forall t' \in T_{invest} \tag{5}$$

The objective function (1) minimizes the sum of capital expenditures ($CAPEX_{t'} = c_{t'}^T x_{t'}$), as defined by Eq. (23) in Part I, and operating expenditures ($OPEX_{t,s} = d_{t,s}^T y_{t,s}$) over a set of seasons $t \in T$ and scenarios $s \in S$ with probabilities $\tau_{t,s}$, as defined by Eq. (24) in Part I. The first-stage variables, $x_{t'}$, are binary and involve decisions on a set of investments (N, $|N| = n$) with fixed capacities, which are allowed in certain time periods, T_{invest} (in our case at the beginning of each year), as described in Eq. (5). Eq. (2) specifies the restrictions on the investment decisions $x_{t'}$, such as constraints (17) and (20) from Part I, which allow certain investments (new major equipment and equipment upgrades) to be executed only once over the time horizon. Eq. (3) summarizes the linking constraints between investment decisions $x_{t'}$ and operational second-stage decisions $y_{t,s}$, which include constraints (15) and (16) for equipment upgrades, constraints (18) and (19) for new equipment and constraint (21) for new storage tanks from Part I. In Eq. (4), $Y_{t,s}$ summarizes the operational constraints (2)–(14) from Part I for season t and scenario s, in which the variables for modes $y_{t,s}^m$ and transitions $y_{t,s}^{tr}$ are binary, and the variables for internal flowrates, inventories, sales and external product purchases, $y_{t,s}^c$, are continuous:

$$Y_{t,s} = \{y_{t,s} = (y_{t,s}^m, y_{t,s}^{tr}, y_{t,s}^c)^T, y_{t,s}^m \in \{0,1\}, y_{t,s}^{tr} \in \{0,1\}, \\ y_{t,s}^c \geq 0 : B_{t,s} y_{t,s} \leq b_{t,s}\} \tag{6}$$

While the index for hours h is omitted in (6), we would like to highlight that each set of operational constraints $Y_{t,s}$ represents a weekly scheduling problem with an hourly time discretization (168 h). Note that the operational problems become independent of each other, once the investment decisions $x_{t'}$ are fixed since there is no inventory carry-over between adjacent seasons or scenarios, which we can exploit with our decomposition strategy in the following section.

3. Bi-level decomposition algorithm

The bi-level decomposition algorithm tackles the original problem (P) by alternately solving a *relaxation* and a *restriction* of P. The *relaxation* of P, denoted as aggregated problem (AP), is built with a subset of the original primal constraints, based on domain-specific knowledge. The idea is that not all primal information is needed in order to determine good values for the complicating variables of P. Once AP is solved, the complicating variables are restricted by either fixing all of them to their respective values obtained from AP, or only fixing the variables that were found to be zero in AP. For the restricted complicating variables, the detailed problem (DP), which is a *restriction* of P, is solved and *primal cuts* that are inferred from DP are added back to AP. The algorithm iterates until the gap between the bounds obtained from AP and DP is within a predefined tolerance. The generic bi-level decomposition algorithm is shown in