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A new decomposition algorithm for multistage stochastic programs with endogenous uncertainties

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ABSTRACT

In this paper, we present a new decomposition algorithm for solving large-scale multistage stochastic programs (MSSPs) with endogenous uncertainties. Instead of dualizing all the initial non-anticipativity constraints (NACs) and removing all the conditional NACs to decompose the problem into scenario subproblems, the basic idea relies on keeping a subset of NACs as explicit constraints in the scenario group subproblems while dualizing or relaxing the rest of the NACs. It is proved that the algorithm provides a dual bound that is at least as tight as the standard approach. Numerical results for process network examples and oilfield development planning problem are presented to illustrate that the proposed decomposition approach yields significant improvement in the dual bound at the root node and reduction in the total computational expense for closing the gap.

1. Introduction

Stochastic programming is typically used to model problems where some of the parameters are random (e.g. uncertain reservoir size, product demand, yields, prices), Birge and Louveaux (1997). In particular, it allows incorporating probability distribution of the uncertain parameters explicitly into the model and provides an opportunity to take corrective actions in the future (recourse) based on the actual outcomes (see Ahmed & Garcia, 2003; Ahmed, Tawarmalani, & Sahinidis, 2004; Clay & Grossmann, 1997; Ierapetritou & Pistikopoulos, 1994; Iyer & Grossmann, 1998; Li & Ierapetritou, 2012; Sahinidis, 2004; Schultz, 2003). Discrete probability distributions of the uncertain parameters that give rise to scenarios are widely considered to represent scenarios that are given by combinations of the realization of the uncertain parameters. Depending on the number of decision stages involved in the model, the stochastic program corresponds to either a two-stage or a multistage problem. The main idea behind two-stage stochastic programming is that we make some decisions (stage 1) here and now based on not knowing the future outcomes of the uncertain parameters, while the rest of the decisions are stage −2 (recourse actions) decisions that are made after uncertainty in those parameters is revealed. In this paper, we focus on more general multistage stochastic programming (MSSP) models where the uncertain parameters are revealed sequentially, i.e. in multiple stages (time periods), and the decision-maker can take corrective actions over a sequence of the stages. In the two-stage and multistage case the cost of the decisions and the expected cost of the recourse actions are optimized.

Based on the type of uncertain parameters involved in the problem, stochastic programming models can be classified into two broad categories (Jonsbraten, 1998): exogenous uncertainty where stochastic processes are independent of decisions that are taken (e.g. demands, prices), and endogenous uncertainty where stochastic processes are affected by these decisions (e.g. reservoir size and its quality). Our decisions can affect the stochastic processes in two different ways (Goel & Grossmann, 2006): either they can alter the probability distributions (type 1) (see Held & Woodruff, 2005; Viswanath, Peeta, & Salman, 2004), or they can determine the timing when uncertainties in the parameters are resolved (type 2) (see Goel, Grossmann, El-Bakry, & Mulkay, 2006; Gupta & Grossmann, 2011). A number of planning problems involving very large investments at an early stage of the project have endogenous (technical) uncertainty (type 2) that dominates the exogenous (market) uncertainty. In such cases, it is essential to incorporate endogenous uncertain parameters while making the investment decisions since it can have a large impact on the overall project profitability. Surprisingly, these problems have received relatively little attention in the literature despite their practical importance.

In this paper, we focus on type 2 of endogenous uncertainty where the decisions are used to gain more information, and resolve

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uncertainty either immediately or in a gradual manner. Therefore, the resulting scenario tree is decision-dependent that requires modeling a superstructure of all possible scenario trees that can occur based on the timing of the decisions (Gupta & Grossmann, 2011). In this context, we present a MSSP framework to model the problems in this class in which special disjunctive constraints with propositional logic are considered to enforce the conditional NACs that define the decision-dependent scenario tree. Recently, few practical applications that involve MSSP with endogenous uncertainty have been addressed: Goel and Grossmann (2004), and Goel et al. (2006) for gas field development planning; Tarhan, Grossmann, and Goel (2009), and Gupta (2013) for oil/gas field investments and operations; Tarhan and Grossmann (2008), and Gupta and Grossmann (2011) for process networks planning; Solak (2007) for project portfolio optimization; Boland, Dumitrescu, and Froyland (2008) for open pit mine scheduling; and Colvin and Maravelias (2008) for pharmaceutical testing.

In general, these MSSPs become very difficult to solve directly as deterministic equivalent since the problem size (constraints and variables) increases with the number of scenarios, whereas the solution time increases exponentially. Therefore, special solution techniques are used to solve problems in this class. Several fullspace approaches for the medium-size problems exploiting the properties of the model and the optimal solution have been proposed. In particular, Colvin and Maravelias (2010) developed a branch and cut framework, while Gupta and Grossmann (2011) proposed a NAC relaxation strategy to solve these MSSP problems under the assumption that only few NACs be active at the optimal solution.

Lagrangean decomposition is a widely used technique to solve large-scale problems that have decomposable structure as in stochastic programs (Caroe & Schultz, 1999; Conejo, Castillo, Minguez, & Garcia-Bertrand, 2006; Fisher, 1985; Guignard, 2003; Ruszczynski, 1997). It addresses problems where a set of constraints links several smaller subproblems. If these constraints are removed by dualizing them, the resulting subproblems can be solved independently. In the case of MSSPs with endogenous uncertainty initial and conditional NACs are the linking constraints, while each subproblem corresponds to the problem for a given scenario. Therefore, the model has a decomposable structure that is amenable to Lagrangean decomposition approaches. In this context, a Lagrangean decomposition algorithm based on dualizing all the initial NACs and relaxing all the conditional NACs that allow parallel solution of the scenario subproblems has been proposed by Gupta and Grossmann (2011). An extended form of this decomposition approach relying on the duality based branch and bound search is also presented in Goel and Grossmann (2006), Tarhan et al. (2009), and Tarhan, Grossmann, and Goel (2011) to close the gap between the UBs and LBs. Solak (2007) used a sample average approximation method for solving the problem in this class, where the sample problems were solved through Lagrangean relaxation and heuristics. However, there are several limitations with these methods including a weak dual bound at the root node, a large number of iterations to converge at each node, and many nodes that may be required during the branch and bound search to close the gap depending on the branching rules and variables. Moreover, the number of subproblems to be solved during each iteration at every node grows linearly with the number of scenarios. In this work, we propose a new decomposition scheme for solving these MSSPs that overcomes some of the limitations of the standard approaches.

The outline of this paper is as follows. First, we introduce the problem statement with particular focus on the problems where timing of uncertainty realization depends on the optimization decisions. Then, a generic mixed-integer linear multistage stochastic disjunctive programming model for endogenous uncertainty problems is presented. Several Lagrangean decomposition approaches that have been used and their limitations are identified next. To overcome these limitations, we propose a new Lagrangean decomposition scheme that relies on the concept of scenario group partitions. Numerical results of process networks and oilfield planning problems with modest number of scenarios are presented for the various decomposition approaches.

2. Problem statement

We focus here on multiperiod planning problems that have endogenous uncertainty in some of the parameters, i.e. where timing of uncertainty realization depends on our decisions. In particular, the time horizon is represented by the discrete set of time periods $T = \{1, 2, \ldots\}$. The set of endogenous uncertain parameters $\Theta = \{\theta_1, \theta_2, \ldots\}$ is considered where each parameter has a discrete set of possible realizations. Therefore, a scenario s represents the possible combination of the realizations of these uncertain parameters with a probability p^s. Note that when some of the parameters θ_p are correlated as they may belong to a particular uncertainty source, then the resulting scenario set will be smaller. The timing of uncertainty resolution in each uncertain parameter depends on the decisions x_t^s (both discrete and continuous) that have been implemented so far. Furthermore, the uncertainty resolution rule can be immediate (Goel & Grossmann, 2006; Gupta & Grossmann, 2011) or gradual (Tarhan et al., 2009) depending on the problem at hand. Therefore, the resulting scenario tree is decision-dependent, and hence we need to use a superstructure of all possible scenario-trees that can occur based on the decisions. In particular, we use logic propositions and disjunctions as in Goel and Grossmann (2006) and Gupta and Grossmann (2011) to represent the scenario-tree for the problems in this class. The uncertainty realizations for each parameter θ_p are assumed to be time invariant. In the next section, we present a MSSP model corresponding to this description.

3. Model

A mixed-integer linear disjunctive MSSP with endogenous uncertainties can be represented in the following compact form:

(MD)

$$\min z = \sum_{s \in S} p^s \sum_{t \in T} c_t x_t^s \tag{1}$$

$$\text{s.t.} \quad \sum_{\tau \le t} A_\tau^s x_\tau^s \le a_t^s \quad \forall t, s \tag{2}$$

$$x_t^s = x_t^{s'} \quad \forall t \in T_I, \quad \forall s, s' \in S \tag{3}$$

$$Z_t^{s,s'} \Leftrightarrow F(x_1^s, x_2^s \ldots x_{t-1}^s) \quad \forall t \in T_C, \forall s, s' \in S \tag{4}$$

$$\begin{bmatrix} Z_t^{s,s'} \\ x_t^s = x_t^{s'} \end{bmatrix} \vee [\neg Z_t^{s,s'}] \quad \forall t \in T_C, \forall s, s' \in S \tag{5}$$

$$x_{jt}^s \in \{0, 1\} \quad \forall t, s, \forall j \in J' \tag{6}$$

$$x_{jt}^s \in R \quad \forall t, s, \forall j \in J \backslash J' \tag{7}$$

The objective function (1) in the above model (MD) minimizes the expectation of an economic criterion over the set of scenarios $s \in S$, and over a set of time periods $t \in T$. For a particular scenario s, inequality (2) represents constraints that govern decisions x_t^s in time period t and link decisions across time periods. The NACs for initial time periods $T_I \subset T$ are given by equations (3) for each scenario pair (s, s') to ensure the same decisions in all the scenarios. The conditional NACs are written for the later time periods $T_C \subset T$