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Minimizing entropy generation in internal flows by adjusting the shape of the cross-section

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ABSTRACT

Entropy generation in fully-developed flow through a duct with heat transfer is discussed. Methods are presented to minimize entropy generation by adjusting the shape of the duct's cross-section. Choosing a different cross-sectional shape allows for control of the competing fluid flow and heat transfer irreversibilities. By controlling the competing irreversibilities, the total entropy generation rate can be minimized. Given the flow rate, heat transfer rate, available cross-sectional area, and the fluid properties, a general design correlation is presented that allows for a determination of the optimal shape of a duct.

1. Introduction

Forced convection heat transfer in a flow passage is affected by two types of losses, namely, loss associated with heat transfer through a temperature difference and loss associated with fluid friction. Entropy generation minimization has been proposed as a criterion for the design of flow passages in internal flow forced convection heat transfer configurations. Because entropy is generated by friction encountered in flowing fluids and by heat transfer through a temperature difference, a calculation of the overall entropy generation allows for an evaluation of these losses on a common scale. Moreover, because the entropy generation is a direct measure of the irreversibilities associated with heat transfer and fluid friction, the overall performance of a device containing heat transfer passages can be improved by calculating and minimizing the total entropy generation of the convective heat transfer process. Numerous studies have shown that in convective heat transfer arrangements the fluid friction and heat transfer losses are coupled, and that attempts to reduce entropy generation associated with heat transfer will increase the entropy generation associated with fluid friction, and vice versa [1,2]. This coupling between fluid flow and heat transfer irreversibilities suggests that the geometry and operating conditions can be optimized to minimize the overall entropy generation.

In many instances, the design engineer is faced with integrating coolant passages into an existing piece of equipment, where the space occupied by the coolant passage is at a premium and the available flow rates may be limited by the size of an existing or a retrofit fan or pump. In these situations, where a coolant passage must be designed so that the cross-sectional area is restricted to some value and where the flow rate through the coolant passage is dictated by the available equipment, one may ask the question: Is there an optimum cross-sectional shape (a circular cross-section, a square, a rectangle, etc.) for the coolant passage that minimizes entropy generation and allows for the best performance?

A number of studies have focused on the calculation and minimization of entropy generation in the fundamental fully-developed convective flow configuration in a duct. In most of these studies the entropy generation in a duct with a particular cross-sectional shape is calculated, and the entropy generation is minimized by adjusting the size (hydraulic diameter or cross-sectional area) of the duct. References can be found where entropy generation is calculated and minimized in ducts with various cross-sectional shapes for laminar and turbulent flow configurations, with constant heat transfer rate per unit length, with constant heat flux, or with constant wall temperature, and in flows with temperature dependent viscosity [3–9].

A few past studies have attempted to compare the entropy generation in ducts with different cross-sectional shapes and to determine the cross-sectional shape that will yield minimum entropy generation [10–12]. Sahin finds that for high Reynolds number flows where fluid friction irreversibility dominates, the optimal shape for a flow channel is the circular shape in both laminar flow with a constant wall temperature [10] and in turbulent flow with constant wall heat flux [11]. Sahin, however, comes to these conclusions after evaluating the entropy generation in a flow of water over rather limited ranges of parameters. Because of the limited parameter space investigated, some questions still remain on the subject of the optimal cross-section for convective heat transfer.

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Nomenclature

A	cross-sectional area
Bo	duty parameter
C_f	coefficient in friction factor
C_h	coefficient in Nusselt number
c_p	specific heat
D_h	hydraulic diameter
$\frac{dT}{dx}$	axial temperature gradient
f	Darcy friction factor
k	thermal conductivity
$\dot{m}$	mass flow rate
Nu	Nusselt number
P	perimeter
Pr	Prandtl number
$\dot{q}'$	heat transfer rate per unit length
R_A	area based Reynolds number
Re	Reynolds number
$\dot{S}'_{gen}$	entropy generation per unit length
T	temperature

Greek

α	exponent in Nusselt number
β	exponent in Nusselt number
γ	exponent in friction factor
μ	viscosity
ρ	density
ϕ	irreversibility distribution ratio
χ	shape factor

Subscripts

opt	at the optimum
min	minimum value

For instance, over what ranges of parameters is the circular cross-section best (to minimize entropy generation)? And, does a general design correlation exist to suggest the shape of an optimal cross-section for minimum entropy generation? To answer these questions, equations are developed here that allow for a determination of the optimal shape of a duct's cross-section given the available cross-sectional area, the heat transfer rate to or from the duct, the flow rate through the duct, and the fluid properties for fully-developed flow.

In Section 2, the equations for entropy generation and the equations describing the geometries that minimize entropy generation in steady-state flow through ducts are presented for both laminar and turbulent fully-developed flow with constant heat transfer rate per unit length. In Section 3, these equations are used to reproduce many of the results that can be found in the previous literature for flows through a duct of specified shape, where entropy generation is minimized by adjusting the size of the cross-section. Also in Section 3, new results are presented for flow in a duct of specified cross-sectional area, where entropy generation is minimized by adjusting the shape of the cross-section. These results are used to show under what circumstances a particular cross-sectional shape will minimize entropy generation. Throughout Section 3, a number of numerical examples are used when discussing results. Finally, conclusions are drawn in Section 4.

2. Entropy generation in steady-state flow through ducts

Consider the general internal flow configuration shown in Fig. 1. Fluid flows through a duct with a cross-sectional area A, a perimeter P, and a hydraulic diameter $D_h = 4A/P$. The shape of the cross-section is arbitrary but constant over the entire length of duct. A single-phase, incompressible and Newtonian fluid flows through the duct with a mass flow rate $\dot{m}$ at a bulk temperature T. Heat is transferred to the duct at a rate (per unit length) of $\dot{q}'$, through the duct wall to the fluid across a temperature difference ΔT. Following Bejan [2], for $\Delta T \ll T$, the entropy generation rate per unit length is given by

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 D_h^2}{4NukAT^2} + \frac{1}{2}\frac{f\dot{m}^3}{\rho^2 A^2 T D_h}, \tag{1}$$

where Nu, f, ρ, and k are the Nusselt number, the Darcy friction factor, the fluid density, and the fluid thermal conductivity, respectively.

Using the same notation as Ratts and Raut [4], the Nusselt number and friction factor for fully-developed laminar or turbulent flow are generalized as

$$Nu = C_h \mathrm{Re}^{\alpha} \mathrm{Pr}^{\beta}, \tag{2}$$
$$f = C_f \mathrm{Re}^{-\gamma}, \tag{3}$$

where $\mathrm{Re} = \dot{m}D_h/A\mu$ is the Reynolds number, Pr is the Prandtl number, and μ is the viscosity. The parameters C_h, C_f, α, β, and γ are tabulated in Table 1 for the circular cross-section and for rectangular and elliptical cross-sections with varying aspect ratios [13]. Additionally, in Table 1 the shape factor, defined as $\chi = P/D_h$ or $\chi = P^2/(4A) = 4A/D_h^2$, is shown for each cross-section. The shape factor is used throughout the following analysis and in the interpretation of results.

After substitution of Eqs. (2) and (3) into Eq. (1), the entropy generation rate is given by

$$\dot{S}'_{gen} = \frac{\dot{q}'^2 D_h^2}{4C_h kAT^2 \mathrm{Re}^{\alpha}\mathrm{Pr}^{\beta}} + \frac{1}{2}\frac{C_f \dot{m}^3}{\rho^2 A^2 T D_h \mathrm{Re}^{\gamma}}. \tag{4}$$

2.1. Ducts with specified cross-sectional shape

First, consider the entropy generation in a duct while holding constant the flow rate, the heat transfer rate, and the fluid properties. Assume that the channel has a specified cross-sectional shape; that is, χ is specified. Entropy generation can then be minimized by choosing the optimum cross-sectional size for the duct.

For any duct with a specified shape factor, the size is determined by either the hydraulic diameter or the cross-sectional area, since these parameters are related through the shape factor $\chi = 4A/D_h^2$. Furthermore, the definition of the shape factor is used to write

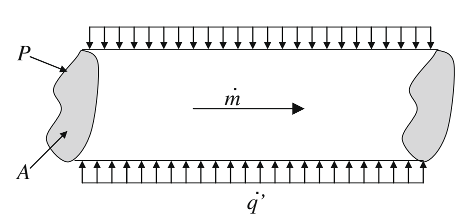

Fig. 1. The general flow and heat transfer configuration.