International Journal of Heat and Mass Transfer 54 (2011) 3811–3821

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Brinkman–Forchheimer modeling for porous media thermoacoustic system

Syeda Humaira Tasnim [a], Shohel Mahmud [b], Roydon Andrew Fraser [a], Ioan Pop [c,*]

[a] Department of Mechanical and Mechatronics Engineering, University of Waterloo, Waterloo, ON, Canada
[b] Mechanical Engineering Program, School of Engineering, University of Guelph, Guelph, ON, Canada
[c] Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania

ARTICLE INFO

Article history:
Received 22 April 2011
Received in revised form 28 April 2011

Keywords:
Brinkman–Forchheimer model
Thermoacoustics
Porous medium
Standing wave

ABSTRACT

In this paper, analytical studies have been conducted on the flow and thermal fields of unsteady compressible viscous oscillating flow through channels filled with porous media representing stacks in thermoacoustic systems. The flow in the porous material is described by the Brinkman–Forchheimer–extended Darcy model. Analytical expressions for oscillating velocity, temperature, and energy flux density are obtained after linearizing and solving the governing differential equations with long wave, short stack, and small amplitude oscillation approximations. Experimental work is also conducted to verify the temperature difference obtained across the porous stack ends. To produce the experimental results, a thermoacoustic heat pump is designed and constructed where reticulated vitreous carbon (RVC) is used as the stack material. A very good agreement is obtained between the modeling and the experimental results. The expression of temperature difference across the stack ends obtained in the present study is also compared with the existing thermoacoustic literature. The proposed expression surpasses the existing expression available in the literature. The system of equations developed in the present study is a helpful tool for thermal engineers and physicist to design porous stacks for thermoacoustic devices.

1. Introduction

Thermoacoustic devices are energy conversion devices that convert heat to acoustic energy and vice versa. A typical thermoacoustic device consists of a resonator, two heat exchangers, and a stack. The stack supports a longitudinal temperature gradient and is the element in which the desired interchange between thermal and acoustic energies takes place. It is sandwiched between heat exchangers which either supply or remove heat from the stack. Installation of heat exchangers near the stack ends provides an opportunity to supply heat flux inside the resonator (through the cold heat exchanger) and extract the heat flux from the resonator (through the hot heat exchanger) in thermoacoustic heat pumps and refrigerators. Thus a thermoacoustic heat pump can be used to heat the other systems which are connected to the hot heat exchanger, and a thermoacoustic refrigerator can be used to cool the systems which are connected to the cold heat exchanger.

Even though thermoacoustic devices have some inherent benefits; e.g., simplicity, reliability, very long life, and environmental friendliness; they are not very efficient (efficiencies are 20% to 30% of Carnot efficiency) and have low power density when compared with typical compression refrigerators. Several authors have considered different approaches to increase the efficiency (Swift [1]), or power density (Poese et al. [2], Bösel et al. [3], and Tijani [4]) of thermoacoustic devices. Poese et al. [2] have designed a thermoacoustic refrigerator to operate at larger pressure amplitudes, since power density in a thermoacoustic device is proportional to the square of the acoustic Mach number (Swift [5]). Another approach to increase the performance of a thermoacoustic device is to decrease the Prandtl number. Viscous friction has a negative effect on the performance of thermoacoustic devices (Swift [5], Tijani [4]). Lower values of Prandtl number can be obtained by using gas mixtures of heavy and light monatomic gases. Tijani [4] showed experimentally that decreasing the Prandtl number (Pr) to 0.2 leads to a coefficient of performance relative to Carnot which is 70% higher than with pure helium (Pr = 0.7). Mahmud and Fraser [6] have examined a new concept of a thermoacoustic system under the influence of a magnetic force. The influence of a magnetic force on the performance of any particular thermoacoustic system has been analyzed using entropy generation analysis.

A different approach to increase the power density of a thermoacoustic device is to use stacks of different arrangements (Bösel et al. [3]) and geometries (Adeff et al. [7]). The stack is the structural heart of a thermoacoustic device. To operate, the stack must conduct thermal energy in a direction perpendicular to the sound wave direction. However, the stack inherently also experiences a finite temperature difference along its length resulting in heat transfer parallel to the sound waves from the hot to the cold heat exchangers. This is an inherent loss mechanism in a stack. In order to reduce this diffusion loss and thereby to enhance the perfor-

* Corresponding author.
E-mail address: popm.ioan@yahoo.co.uk (I. Pop).

0017-9310/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2011.05.004

Nomenclature

a_0	a complex parameter, = $(1+i)\sqrt{1+\phi/(2iDa)}/\delta_v$
b_0	a complex parameter, = $(1+i)\sqrt{\sigma}/\delta_k$
C_p	specific heat of the fluid at constant pressure, J kg^{-1} K^{-1}
Da	Darcy number, = K/δ_v^2
d	pore diameter, m
$\dot{\mathbf{E}}$	energy flux density vector, W m^{-2}
$\dot{E}_2$	second order energy flux density, W m^{-2}
E_2	global energy flux, W
F	Forchheimer's coefficient
f	frequency of oscillation, Hz
f_v	Swift's function of thermoacoustics, = tanh $(a_0y_0)/(a_0y_0)$
$\tilde{f}_v$	complex conjugate of f_v
f_k	Swift's function of thermoacoustics, = tanh $(b_0y_0)/(b_0y_0)$
$\tilde{f}_k$	complex conjugate of f_k
i	complex number, = $\sqrt{-1}$
h	enthalpy, J kg^{-1}
K	permeability of the porous medium, m^2
k	thermal conductivity, W m^{-1} K^{-1}
k_0	inverse of the wave number, = ω/c
L_s	length of the stack, m
p	pressure, N m^{-2}
Pr	Prandtl number of the fluid, = δ_v^2/δ_k^2
S_w	Swift number, = y_0/δ_v
t	time, s
T	temperature of the fluid, °C
u	axial component of the fluctuating velocity inside the channel, m s^{-1}
U	dimensionless axial velocity (see Eq. (35))
u_0	a reference velocity, m s^{-1}
v	transverse velocity, m s^{-1}
$\mathbf{V}$, $\mathbf{v}$	velocity vector, m s^{-1}
W_2	total work flux, W
x	axial distance, m
y	transverse distance in fluid, m
Y	dimensionless transverse distance in the porous medium, = y/y_0
y_0	half width of a channel, m

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	thermal expansion coefficient, K^{-1}
γ	isobaric to isochoric specific heat ratio, = C_p/C_v
Γ_0	temperature gradient ratio in the presence of a porous medium, = $(T_{ad}/T_{sw})\,[1+\phi/(2iDa)]$
δ_v	viscous penetration depth, = $\sqrt{2\nu/\omega}$
δ_k	thermal penetration depth, = $\sqrt{2\alpha_f/\omega}$
∇T_m	mean temperature gradient, = $\partial T_m/\partial x$
∇p_1	pressure gradient, = $\partial p_1/\partial x$
∇T_{cr}	critical temperature gradient, °C m^{-1}
μ	dynamic viscosity of the fluid, N m^{-2} s
ν	kinematic viscosity, m^2 s^{-1}
$\boldsymbol{\sigma}$	viscous stress tensor, N m^{-2}
σ	heat capacity ratio, = $\phi+(1-\phi)(\rho_{sm}\,C_{sm})/(\rho C_p)$
ω	circular frequency, rad s^{-1}
ρ	density of the fluid, kg m^{-3}
τ	time period, = $2\pi/\omega$
Ψ	a dimensionless parameter, = $\phi(2iDa)^{-1}+\frac{2Fu_0\phi}{i\omega\sqrt{K}}$
λ	wavelength, m
Π	width of the plate, m
ϕ	porosity of the porous medium, = void volume/total volume

Subscripts and Superscripts

0	reference value
1	first order variable
av	average value
f	properties of the fluid
m	mean value
sm	properties of the solid matrix material in the porous medium
w	value at wall

Symbols

$\Re$[ ]	real part of an expression
$\Im$[ ]	imaginary part of an expression

mance of a thermoacoustic device; Bösel et al. [3] have proposed an alternate stack arrangement. Bösel et al.'s approach [3] is to replace the typical parallel plate stacks that are much longer than the acoustic displacement amplitude, by parallel plate segments which are only a fraction of the displacement amplitude long and randomly oriented to each other. This simplified stack arrangement is expected to benefit from improved heat transfer characteristics and an anisotropic thermal conductivity (Bösel et al. [3]).

Adeff et al. [7] have used porous RVC as a stack material in thermoacoustic devices. RVC is a rigid glassy carbon material, with a porous sponge like structure, a large surface area, and a low thermal conductivity. Based on the experimental measurements, Adeff et al. [7] shows that RVC has a definite potential as a low cost, easy to fabricate material suitable for use in all types of thermoacoustic devices.

Recently, Mahmud and Fraser [8] and Tasnim and Fraser [9] have presented investigations that consider the stack plate to be composed of multiple parallel plates and the space between the stack plates is occupied by porous materials. For a steady state case, Hunt and Tien have showed experimentally [10] that the porous medium enhances heat transfer from a surface as compared to predicted results for slug or for laminar flow in a channel. Mahmud and Fraser [8] consider a simplification of Vafai and Tien's porous medium model [11] for the modeling of momentum equation. Mahmud and Fraser's work [8] show that if a porous medium is used in between the thin parallel plates, much more efficient thermoacoustic engine designs are possible in the Da (= K/δ_v^2, where K is the permeability of the porous medium and δ_v is the viscous penetration depth [5]) range 0.1–1.0 and for S_w (non-dimensional stack plate spacing) $\geqslant 2$. Tasnim and Fraser [9] have presented an analytical investigation considering Darcy flow model for the modeling of momentum equation. The thickness of the channel wall is considered to be nonzero in Tasnim and Fraser [9] and the entire problem is treated as a conjugate heat transfer problem. Tasnim and Fraser [9] have showed that refrigeration performance comparable to that of a traditional plastic parallel plate stack is achievable using RVC foam (ϕ = 0.95, Lc_k = 2.11, $Lc_k = y_0/\delta_k$, where y_0 and δ_k are the half width of the stack plate and the thermal penetration depth [5], respectively) as a porous medium.

A thermoacoustic theory for a bulk porous medium is developed by Roh et al. [12] based on parallel, capillary-tube-based theories. The authors have introduced the tortuosity, the viscous dynamic shape factor, and the thermal dynamic shape factor to extend thermoacoustic theory to a bulk porous medium. Comparisons of thermoacoustic properties (of RVC and aluminum foam developed by Roh et al. [12]) show good agreement with experiment.

A simplified model based on steady-flow correlations and Lagrangian approach is developed by Jung and Matveev [13] for thermoacoustic analysis of short transverse-pin stacks with moderate temperature differences. This method provides fast