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Analytical solution of forced convective heat transfer in tubes partially filled with metallic foam using the two-equation model

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ABSTRACT

In this study, an analytical solution for fully developed forced convection in a tube partially filled with open-celled metallic foams is presented. In the foam region, the Brinkman flow model is used to describe the fluid transport, and the local thermal non-equilibrium model is adopted to represent the fluid–solid energy exchange. At the foam–fluid interface, interfacial coupling conditions for temperature are proposed and used to derive the analytical solution. Velocity and temperature profiles are derived from this solution, and explicit expressions for the friction factor and the Nusselt number are obtained. A parametric study is conducted to study the influences of various factors on flow resistance and heat transfer performance. The present analytical solution establishes a benchmark for similar work hereafter.

1. Introduction

As a type of promising porous medium, metallic foam has excellent heat transfer performance due to its considerable advantages of large specific area, high solid thermal conductivity, and strong flow-mixing capability. Using metallic foams for heat transfer enhancement is a novel method. Metallic foams have great potential in heat-transfer-related applications, such as chemical reactors, heat exchangers, heat sinks, reformers, cooling of nuclear reactors, energy storage, and others. Among these, efficient and compact heat exchangers and heat sinks are needed in many engineering applications, which can motivate corresponding research interest.

Flow and heat transfer in a duct fully filled with metallic foams have been extensively investigated [1–8]. Calmidi and Mahajan [1] and Zhao et al. [2] presented experimental and numerical studies on forced convection in metallic foams. Lu and Zhao et al. [3,4] analytically investigated the metallic foam filled tube and annulus with the Brinkman–Darcy model. Dukhan and Chen [5] conducted analytical and experimental study on forced convective heat transfer in a parallel-plate channel fully filled with metallic foams. DeGroot et al. [6] undertaken a numerical study to explore details of forced convection in finned aluminum foam heat sinks. Du et al. [7] performed numerical study on heat transfer in metal-foam double-pipe heat exchangers with parallel-flow arrangement. Mahjoob and Vafai [8] summarized different flow and heat transfer models for metallic foams appearing in existing literatures. Overall, it is widely accepted that the metallic foams can greatly enhance heat transfer but at the expense of large pressure drop. A feasible way to lower the flow resistance is to partially fill the duct with metallic foams.

Forced convection in a duct partially filled with metallic foams is a potential method for heat transfer enhancement. Convective mechanisms are usually affected by interactions at the foam–fluid interface, which offer some fundamental scientific problems. The flow and heat transfer characteristics of the two regions separated by the interface are significantly different. The velocity and temperature fields should be solved separately from interfacial coupling conditions. For forced convection in a configuration fully filled with porous media, Mahjoob and Vafai [9] conducted a comprehensive analysis on the thermal-transport process through double-layered and multilayered biological porous media, and the blood and tissue temperatures were obtained using the Darcy equation. Yang and Vafai [10] analytically investigated the temperature-gradient bifurcation of forced convection in porous medium-filled parallel-plate channels with heat generation under condition of uniform cross-sectional velocity. However, for forced convection in a duct partially filled with a porous medium, only a few analytical solutions have been reported for specific cases [11–13]. Vafai and Thiyagaraja [11] analyzed fluid flow and heat transfer at the porous matrix–fluid interface and derived a perturbation solution. Poulikakos and Kazmierczak [12] presented an analytical solution for forced convection in a duct partially filled with porous materials. Chikh et al. [13] obtained an analytical solution for non-Darcian convection in an annulus partially filled with a porous medium. For all these analytical studies, the local thermal equilibrium (LTE) model has been adopted, in which the temperatures of fluid and solid are considered to have the same values.

For heat transfer modeling in porous media, the local thermal non-equilibrium (LTNE) model is an alternative method. Lee and

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Nomenclature

a_{sf}	specific surface area, m^{-1}
A	area, m^2
c	specific heat, J kg^{-1} K^{-1}
d_f	fiber diameter, m, $d_f = 1.18\sqrt{(1-\varepsilon)/(3\pi)}$ $(1-\exp((\varepsilon-1)/0.04))$
d_p	pore diameter, m, $d_p = 0.0254/\omega$
Da	Darcy number
f	friction factor
h	heat transfer coefficient, W m^{-2} K^{-1}
h_{sf}	local convective heat transfer coefficient, W m^{-2} K^{-1}
K	permeability, m^2
k	thermal conductivity, W m^{-1} K^{-1}
Nu	Nusselt number
p	pressure, N m^{-2}
P	dimensionless pressure drop
Pr	Prandtl number
q	heat flux, W m^{-2}
r	radius, m
r_i	interfacial radius for the foam–fluid interface
r_0	tube radius, m
R	dimensionless radius
Re	Reynolds number, $Re = \rho_f u \cdot 2r_0/\mu_f$
Re_d	local Reynolds number for flow around a cylinder, $Re_d = \rho_f u d_f/\mu_f$
Re_K	permeability Reynolds number, $Re_K = \rho_f u\sqrt{K}/\mu_f$
R_i	dimensionless interfacial radius $R_i = r_i/r_0$
s	dimensionless factor
t	dimensionless factor
T	temperature, K
u	velocity, m s^{-1}
u_m	mean velocity, m s^{-1}
U	dimensionless velocity
x	axial position, m

Greek symbols

ε	porosity
θ	dimensionless temperature
μ	dynamic viscosity, kg m^{-1} s^{-1}
ρ	density, kg m^{-3}
φ	polar angle, rad
ω	pore density, PPI (pores per inch)

Subscripts

b	bulk
e	effective
f	fluid/fiber
i	interface
m	mean
s	solid
w	wall
x	x position

Vafai [14] presented an error map for the LTE and LTNE models. It was noted that the LTE model is not accurate when the thermal conductivity between the fluid and the solid matrix is significant, as is the case for metallic foams. However, analytical solutions using the two-equation model for tube partially filled with metallic foam are not available because of the two challenges: extension of the number of variables and equations, and solid–fluid interfacial closure conditions that occur during derivation of two-equation model solutions.

In this study, a mathematical model for fully developed forced convective heat transfer in a tube partially filled with metallic foam is proposed by adopting the Brinkman–Darcy and the two-equation model. Furthermore, analytical solutions are derived, in which the interfacial coupling conditions corresponding to the LTNE model are employed. Detailed information regarding the procedure for solving this problem is presented. Velocity and temperature profiles are plotted from the analytical solution, and explicit expressions for the Nu number and the friction factor are obtained. A parametric study is implemented and discussed further in depth, with particular attention paid for flow resistance and heat transfer performance.

2. Mathematical model

The geometric configuration of the problem is shown in Fig. 1. An annular metallic-foam layer is attached to the inner wall of the tube. The interface between the foam region and the foam-free region within the tube is concentric. Constant heat flux is imposed on the wall of the tube. The wall radius is r_0 and the interface diameter is r_i. The single-phase fluid is assumed laminar, with constant thermal properties in the fully developed region. The fluid flows through the tube, absorbing heat from the tube wall and the foam ligaments. Metallic foams are both homogeneous and isotropic, with negligible thermal dispersion, because of their high thermal conductivity, as indicated by Calmidi and Mahajan [1].

2.1. Governing equations

There are two sets of governing equations corresponding to the foam region and the foam-free region. In the foam region, the Brinkman-extended Darcy equation is adopted as the momentum equation, and the two-equation model accounting for the LTNE effect is used to establish the energy equations. The commonly used angle brackets for the volume-averaged variables, such as u, v, p, T_s, and T_f, are dropped for simplification of presentation. Based on these assumptions, the following conditions hold for fully developed flow and temperature fields in the foam region and the foam-free region

$$\frac{\partial u}{\partial x} = 0, \quad v = 0, \quad \frac{\mathrm{d}p}{\mathrm{d}x} = \text{constant}, \tag{1}$$

$$\frac{\partial}{\partial x}\left(\frac{T_f - T_w}{T_{f,b} - T_w}\right) = 0, \quad \frac{\partial}{\partial x}\left(\frac{T_s - T_w}{T_{f,b} - T_w}\right) = 0, \tag{2}$$

$$\frac{\mathrm{d}T_{f,b}}{\mathrm{d}x} = \frac{\mathrm{d}T_w}{\mathrm{d}x} = \text{constant}, \quad h_x = \text{constant}. \tag{3}$$

Eq. (2) can be rewritten as

$$\frac{\partial T_f}{\partial x} = \frac{\mathrm{d}T_w}{\mathrm{d}x} + \left(\frac{T_f - T_w}{T_{f,b} - T_w}\right)\frac{\mathrm{d}T_{f,b}}{\mathrm{d}x} - \left(\frac{T_f - T_w}{T_{f,b} - T_w}\right)\frac{\mathrm{d}T_w}{\mathrm{d}x}, \tag{4}$$

$$\frac{\partial T_s}{\partial x} = \frac{\mathrm{d}T_w}{\mathrm{d}x} + \left(\frac{T_s - T_w}{T_{f,b} - T_w}\right)\frac{\mathrm{d}T_{f,b}}{\mathrm{d}x} - \left(\frac{T_s - T_w}{T_{f,b} - T_w}\right)\frac{\mathrm{d}T_w}{\mathrm{d}x}. \tag{5}$$

After substituting Eq. (3) into Eqs. (4) and (5), Eq. (6) is obtained

$$\frac{\mathrm{d}T_w}{\mathrm{d}x} = \frac{\mathrm{d}T_{f,b}}{\mathrm{d}x} = \frac{\partial T_f}{\partial x} = \frac{\partial T_s}{\partial x} = \text{constant}. \tag{6}$$

Thus, the governing equations can be simplified as follows: