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# Effect of temperature-dependent consistency index on the exiting sheet thickness in the calendering of power-law fluids

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ABSTRACT

In this work we treat theoretically the calendering process of inelastic (power-law fluid) sheets of finite initial thickness, taking into account that the consistency index of the fluid is a well-defined function of the temperature. We predict the influence of the temperature-dependent consistency index on the exiting sheet thickness in the calendering process. The mass, momentum and energy balance equations, based on the lubrication theory, were nondimensionalized and solved for the velocity, pressure and temperature fields by using perturbation and numerical techniques, where the exiting sheet thickness represents an eigenvalue of the mathematical problem. When the above variables were obtained, the exiting sheet thickness in the calendering process was determined, considering the influence of the temperature variations in the process. The mentioned governing equations contain four dimensionless parameters: the Graetz number, $Gz$, a geometrical aspect ratio, $\beta$, the power law index of the fluid, $n$, and a parameter that takes into account the effect of the variable consistency index as a function of the temperature and $\epsilon$, defined as the ratio of the Nahme–Griffith number, $Na$, to the Graetz number. Using the limit of $\epsilon \ll 1$, the dimensionless exiting sheet thickness of the calendering process have been obtained as a function of the involved dimensionless parameters. The numerical results show that the inclusion of temperature-dependent consistency index effect modifies the dimensionless exiting sheet thickness in about 6.91%.



## 1. Introduction

The viscous fluid flow through the narrow region between two rotating rolls in such a way as to produce a thin sheet has been extensively studied over the past 50 years. The specialized literature for this calendering process is reviewed in the following paragraphs. The theoretical analysis regarding the above mechanism was developed by Gaskell [1] and McKelvey [2] for isothermal Newtonian and power-law fluids. Zheng and Tanner [3] applied the Phan–Thien–Tanner fluid model for the calendering process of inelastic and viscoelastic sheets using a perturbation method. Sofou and Mitsoulis [4] used the lubrication approximation theory to provide numerical results for isothermal viscoplastic calendering sheets with a desired final thickness. Mitsoulis [5] numerically investigated the shape of the free surfaces of the entering and exiting sheets for the process of calendering viscoplastic sheets with a finite thickness. The combined effects of asymmetry and viscous heating for the non-isothermal nip flow in calendering were considered by Dobbels and Mewis [6]. The effect of viscous dissipation on calendering process of power-law fluids have been studied by Kiparissides and Vlachopoulos [7]. They reported the temperature profiles due to viscous dissipation in calendering gap, finding two maxima in the vicinity of the roll surfaces. In addition, the maximum temperatures exhibit two local maxima and one minimum in the direction of the flow. It appears that the only work about non-isothermal calendering of non-Newtonian fluids is just the aforementioned work. However, in this work they determined the temperature profile, disregarding the influence of the temperature on the final exiting sheet thickness. In this direction, Middleman [8] developed a simple model of the sensitivity of calendered thickness to temperature fluctuations. He concluded that "*a 3° variation in temperature will cause more than 20% variation in calendered thickness!*" In this sense, to our knowledge there are no works that validate the aforementioned sentence. Therefore, the goal of this work is to determine the influence of a temperature rise in the calendering process by viscous dissipation, considering the temperature-dependent consistency index on the exiting sheet thickness.

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**Nomenclature**

| | |
|---|---|
| $a$ | empirical parameter defined in Eq. (5) |
| $c_p$ | heat capacity [J/kg K] |
| $Gz$ | Graetz number |
| $h(\bar{x})$ | distance from center plane to periphery of roll at any value of $\bar{x}$ |
| $H_0$ | one-half of the thickness at the nip [m] |
| $H_f$ | one-half of the incoming sheet thickness [m] |
| $H$ | one-half of the exiting sheet thickness [m] |
| $k$ | thermal conductivity [W/m K] |
| $K_0$ | reference consistency index [Pa s$^n$] |
| $K$ | consistency index [Pa s$^n$] |
| $n$ | power law index |
| $Na$ | Nahme–Griffith number |
| $P$ | dimensionless pressure |
| $\overline{P}$ | pressure in physical units [Pa] |
| $Pe$ | Peclet number |
| $\overline{Q}$ | flow rate, in physical units [m$^3$/s] |
| $Q$ | dimensionless flow rate |
| $R$ | cylinder radius [m] |
| $Re^*$ | modified Reynolds number |
| $T$ | temperature [K] |
| $T_0$ | reference temperature [K] |
| $\Delta T_c$ | characteristic temperature rise [K] |
| $u$, $v$ | dimensionless longitudinal and transversal velocities |
| $U$ | roll speed [m/s] |
| $\bar{u}$, $\bar{v}$ | longitudinal and transversal velocities in physical units [m/s] |
| $\bar{x}, \bar{y}$ | cartesian coordinates |
| $Y$ | dimensionless transversal coordinate, $Y = y/(1+\chi^2)$ |

*Greek letters*

| | |
|---|---|
| $\beta$ | aspect ratio, defined as $\beta = \sqrt{H_0/2R}$ |
| $\epsilon$ | dimensionless parameter, defined as $\epsilon = Na/Gz$ |
| $\eta$ | non-newtonian viscosity |
| $\theta$ | dimensionless temperature of the fluid |
| $\theta_0$ | dimensionless temperature zeroth-order, $\epsilon^0$ |
| $\theta_1$ | dimensionless temperature first order, $\epsilon^1$ |
| $\lambda$ | leave-off distance, defined in Eq. (15) |
| $\lambda_0$ | leave-off distance zeroth-order |
| $\lambda_1$ | leave-off distance first-order |
| $\rho$ | fluid density [kg/m$^3$] |
| $\bar{\tau}_{xy}$ | shear stress in physical units [Pa] |
| $\chi$ | dimensionless longitudinal coordinate |
| $\omega$ | angular velocity |

In order to predict the final sheet thickness on calendering of a power-law fluid, the non-linear momentum and energy balance equations, based on the lubrication approximation, were decoupled by using a regular perturbation technique. It is worth to mention that only the leading order or zeroth-order solution for the mass and momentum equations were solved in previous works [4,8]. In our case, the first-order solution of the above equations is function of the zeroth-order of the energy equation, which was solved by a Crank–Nicolson method. In this form, we can determine the first order of the pressure and velocity profiles, following the same method as that given by Middleman [8].

## 2. Formulation

In Fig. 1a we show the sketch of the physical model studied. Two cylinders separated by a thin film of a polymeric liquid are rotated in the same direction. Each cylinder has a radius $R$, rotating with a constant angular velocity, $\omega$, resulting in a linear velocity at its surface, given by $U = \omega R$. The minimum gap half-height, $H_0$, is such that $H_0 \ll R$ and the one-half exiting sheet thickness is represented by $H$. The geometry of the roll surface is given as $h(\bar{x}) = H_0(1 + \bar{x}^2/2RH_0)$[8]. The roll surfaces are found at a constant temperature, $T_0$. The location $\bar{x}$ where the sheet first bites the rolls is here represented by $-\bar{x}_f$, which is known. On the other hand, the actual leave-off distance location of the sheet, represented by $\lambda$, is unknown and must be determined in the present analysis. Due to the symmetry of the physical model, we consider only for convenience, the upper side of this configuration. Therefore, we select the origin of the coordinate system as shown in Fig. 1a, where axis $\bar{y}$ points up, i.e., in the opposite direction of the gravity vector, and the positive $\bar{x}$ axial axis points in the direction of the flow. Since many materials used in calendering are frequently non-Newtonian, we use the well known power-law model, including the temperature-dependent consistency index.

The mass, momentum and energy equations in steady state are the following:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{1}$$

$$\rho\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \rho\bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{\partial \overline{P}}{\partial \bar{x}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}}, \tag{2}$$

$$\rho\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \rho\bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{\partial \overline{P}}{\partial \bar{y}} + \frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}}, \tag{3}$$

$$\rho c_p\left(\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}}\right) = k\left(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2}\right) + \bar{\tau}_{xy}\frac{\partial \bar{u}}{\partial \bar{y}}, \tag{4}$$

where the shear stress is given as [9],

$$\bar{\tau}_{xy} = K_0 \exp[-a(T - T_0)]\left|\frac{\partial \bar{u}}{\partial \bar{y}}\right|^{n-1}\frac{\partial \bar{u}}{\partial \bar{y}}. \tag{5}$$

The boundary and initial conditions associated with Eqs. (1)–(4) are:

$$\bar{y} = 0 : \frac{\partial \bar{u}}{\partial \bar{y}} = 0, \tag{6}$$

$$\bar{y} = h(\bar{x}) : \bar{u} = U, \tag{7}$$

$$\bar{y} = 0 : \frac{\partial T}{\partial \bar{y}} = 0, \tag{8}$$

$$\bar{y} = h(\bar{x}) : T = T_0, \tag{9}$$

$$\bar{x} = -\bar{x}_f : T = T_0. \tag{10}$$

In the region of the nip and extending to either side (i. e., in the $\pm\bar{x}$ direction) by a distance of the order of $\bar{x}_0$ (see Fig. 1a), the roll surface are nearly parallel for $H_0 \ll R$. Then, we can assume that the flow is nearly parallel, as we shown lines below. According to this assumption, the boundary conditions (6)–(10) are those used as in the lubrication approximation theory.

In the above equations, $\bar{u}$, $\bar{v}$, $\overline{P}$ and $T$ represent the velocity components in $\bar{x}$ and $\bar{y}$ directions, pressure and temperature fields of the fluid, respectively. $\rho$, $c_p$, $k$, $K_0$ are the density, heat capacity, thermal conductivity and the consistency index of the polymeric liquid evaluated at a reference temperature $T_0$, respectively. In