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A fast modelling tool for plate heat exchangers based on depth-averaged equations

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ABSTRACT

In this study, the depth-averaged flow and energy equations for plate heat exchangers are presented. The equations are derived by integrating the original 3D flow and energy equations over the height of the gap between the bottom and top plates. This approach reduces the equations from 3D to 2D but still takes into account the frictions on the surfaces and heat transfer through the plates. The depth-averaging reduces the elapsed time of CFD simulations from hours to minutes. Thus, it is very practicable modelling method in real time design work. 2D CFD simulations with depth-averaged equations are compared with full 3D models for five different corrugation angles and corrugation lengths. The simulation results show that the 2D model predicts with relatively good accuracy the profile of the pressure drop and the temperature change as a function of the corrugation angle and the function of the corrugation length. In order to get more extensive information about the significance of the different geometry parameters on the efficiency of the heat exchanger, we simulated 30 different geometries with the fast 2D model. The results suggest that the temperature change is not as sensitive for the geometrical modifications as the pressure drop.

1. Introduction

Increasing needs to reduce energy costs emphasize the efficiency aspects in the heat exchanger design process. The most common type of heat exchangers are plate heat exchangers, in which plates are used to separate the hot and cold fluids. Thus, the design of the plate surface is the key issue when aiming to maximize the heat transfer. Nevertheless, the pressure drop has to be taken into account as well because it is one important factor affecting energy losses when pumping water in the heat exchanger. Both the pressure drop and the heat transfer coefficient are determined by the flow conditions, especially turbulence.

Most of the recent research on heat exchangers has focused on different correlations between the Nusselt number and the friction factor, because these quantities can be determined experimentally over the whole heat exchanger, i.e., globally [7]. The pressure drop and temperature change for multiple gap plate heat exchangers depend partly on the flow distribution to the channels from the conduit. The equations describing the flow distribution and pressure drop in plate heat exchangers have been derived analytically [2,3]. These equations agree well with experimental results for small and large plate packages, which shows how the pressure distribution between the first and the last channel is more uneven for larger plate packages [4,5]. Temperatures, heat loads and pressure drops, and also their distributions using a few channels and flat plates have been analyzed with computational fluid dynamics (CFD) [6]. However, in our research the effect of maldistribution need not be taken into account, because only the dependence of the pressure drop and heat transfer on the channel geometry is studied. Flow phenomena and heat transfer rate have been analyzed globally and locally with CFD based modelling, giving invaluable information about the effect of geometrical details on heat transfer rate and on friction losses for the design work [8]. The small scale effects of corrugation on heat transfer rate and friction losses have been studied by simulating a few corrugation cycles using the side profile of the channel [9].

However, when determining the dependence of the heat transfer rate and pressure losses on geometry parameters in the product development process, efficiency analysis of a whole single plate with dozens of different parameter combinations is needed. In addition, the pre-processing, i.e., generating geometry and mesh, is a very complicated and time consuming part of the whole development process, because it has to be done for every geometry. Therefore, simplifications are needed in order to make modelling faster for a real design process. This can be accomplished by using depth-averaged equations for flow and heat transfer modelling. These equations are derived by integrating the original 3D flow and energy equations over the height of the gap between the bottom and top plates. This approach reduces the equations from 3D to 2D but still takes into account the frictions on the surfaces, height of the channel and heat transfer through the plates. Even though some of the 3D flow structures are lost, this approach

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Nomenclature

D	channel's depth	S	velocity profile
S_m	source term of mass	c_f	bed friction coefficient
P_{kv}	source of turbulence kinetic energy	u_f	friction velocity
$P_{\epsilon v}$	source of dissipation	e	specific internal energy
γ	expansion coefficient of plate area	P	at point P
y^*	non-dimensional distance	Pr_t	turbulent Prandtl number
κ	von Kármán constant	E	empirical constant
y_T^*	non-dimensional thermal sublayer thickness	y_P	distance from point P to the wall
Δp	pressure drop	ΔT^*	normalized temperature change
$\overline{\Delta p}$	average of pressure drops	$\overline{\Delta T}$	average of temperature changes
$\Delta p*$	normalized pressure drop		

describes the phenomena in the heat exchanger with relatively good accuracy and it can be used in real time design work.

This paper introduces the depth-averaged flow and energy equations for a plate heat exchanger. The goal of this approach is to reduce the elapsed time of CFD simulations from hours to minutes. The source terms for turbulence kinetic energy and its dissipation rate in the standard $k-\epsilon$ model are taken from [10]. While they specified the heat transfer coefficient empirically as a global constant, we determine it in every computational cell by using local flow properties. The equations are used for modelling the chevron-type heat exchanger, where the plate geometries are presented with sine functions that contain the most important geometry parameters: the corrugation angle, the corrugation length and the corrugation depth. With this approach we need to generate only one 2D mesh and can present the height of the plate gap in every horizontal position with the source terms in governing equations. Thus, we can skip geometry pre-processing work and need only change the parameters of the sine functions. In addition, modelling is much faster with a 2D mesh than with a 3D mesh.

2. Theory

2.1. Flow equations

In the present study, heat exchanger modelling is done using depth averaged flow and energy equations with the commercial CFD software Fluent. The steady-state depth-averaged mass conservation law can be written as

$$\nabla\cdot(\rho\vec{u}) = -\frac{\rho}{D}\vec{u}\cdot\nabla D = S_m \tag{1}$$

in which D is the channel's depth, $\vec{u} = (\overline{u},\overline{v})$ is the depth-averaged velocity vector and ρ is the density. The right hand side is defined as a source term S_m. The source term of mass appears also in the momentum equation that is required for flow modelling. Depth-averaging of the z-directional derivatives of the stress tensor in the 3D momentum equation leads to the following expression

$$\frac{1}{D}\int_0^D \frac{\partial}{\partial z}(\tau_{iz})dz = \frac{1}{D}[\tau_{iz}(D)-\tau_{iz}(0)], \quad i = x,y, \tag{2}$$

where τ_{xz} and τ_{yz} are the friction forces on the bottom and the upper surfaces. Fluid is assumed to be flowing horizontally, that is, the velocity component $w = 0$. Horizontal velocity components u and v are defined with the help of the average velocities $\overline{u}$ and $\overline{v}$, and the z-directional profile S in the following form

$$u(x,y,z) = \overline{u}(x,y)\cdot S(z), \tag{3}$$
$$v(x,y,z) = \overline{v}(x,y)\cdot S(z), \tag{4}$$

where

$$S(z) = \frac{n+1}{n}\left(1-\left|1-\frac{2}{D}z\right|^n\right), \tag{5}$$

where $n = 7$ for turbulent flow. Therefore, the friction forces on the bottom and the top surfaces are

$$\tau_{xz} = \mu\frac{\partial u}{\partial z} = \mu\overline{u}\frac{\partial S}{\partial z} = -\frac{4\mu\overline{u}(n+1)}{D^2}, \tag{6}$$

$$\tau_{yz} = \mu\frac{\partial v}{\partial z} = \mu\overline{v}\frac{\partial S}{\partial z} = -\frac{4\mu\overline{v}(n+1)}{D^2}, \tag{7}$$

where μ is the dynamic viscosity. Now the 2D depth-averaged conservation law-of-momentum without the time derivative term can be written as

$$\nabla\cdot(\rho\vec{u}\vec{u}) = -\nabla p + \nabla\cdot(\overline{\overline{\tau}}) + \vec{u}S_m - \frac{4\mu\vec{u}(n+1)}{D^2}, \tag{8}$$

where p is the pressure and $\overline{\overline{\tau}}$ is the stress tensor.

Derived mass balance and momentum equations are sufficient for laminar flow cases, otherwise we need to simulate turbulence quantities. The simplified production terms for turbulence and its dissipation are from [10]. The production of turbulent energy due to wall friction for k and ϵ equations is included via the production terms

$$P_{kv} = \rho c_k\frac{u_f^3}{D}, \tag{9}$$

$$P_{\epsilon v} = \rho c_\epsilon\frac{u_f^4}{D^2}, \tag{10}$$

where the empirical parameters are

$$c_k = \frac{1}{c_f^{1/2}},$$

$$c_\epsilon = 3.6\frac{c_{2\epsilon}c_\mu^{1/2}}{c_f^{3/4}},$$

where c_f

$$c_f = \frac{u_f^2}{u^2} \tag{11}$$

is the friction coefficient, u_f is the friction velocity, u is the velocity, and the constants for standard $k-\epsilon$ turbulence model are $c_\mu = 0.09$ and $c_{2\epsilon} = 1.92$. Now we have the source terms for mass balance, momentum equations and turbulence model. These sources are used in simulating 3D channel flow with 2D mesh. In addition, when modelling the behavior of plate heat exchangers we also need the energy equation for temperature modelling, which is derived in the next section.