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Microbubble or pendant drop control described by a general phase diagram

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ABSTRACT

Controlling states and growths of a microscale bubble (or pendant drop) in a static liquid on a solid surface (or orifice) can be achieved by introducing general dimensionless phase diagrams provided in this work. Nowadays, microbubbles are often used to control transport phenomena in various micro- and nano-technologies. This work parametrically presents general three-dimensional phase diagrams of a microbubble on a solid surface by applying perturbation solutions with accuracy to the second power of Bond number of Young–Laplace equation in the literature. The phase diagrams are found to be divided into three regions, depending on if the microbubble surface contains the inflection point or neck. The general growth, departure and entrapment of a microbubble thus can be described by path lines on diagrams by adjusting two of three dimensionless parameters governing the apex and base radii, and contact angle to satisfy the desired requirement. The initial condition is Bond number, defined as the ratio of hydrostatic and capillary pressures at an initial or critical state. Validity of this model is confirmed by comparing with available theoretical data, agreed with experimental results in the literature.

1. Introduction

The kinetics of nucleation, growth, departure, or merger of bubbles due to overheat, supersaturation, cavitation and electrical potential strongly affect heat, momentum, and mass transport rates in ubiquitous fields, such as nucleate and pool boiling [1], bubble columns intensively utilized as multiphase contactor and reactors in chemical, petrochemical, biochemical and metallurgical industries [2], separator [3], floatation [4], cavitation [5], drag reduction [6], sonoluminescence [7], sonochemistry [8], surface cleaning [9], membrane processes [10], and metallurgical processes for degassing of liquid metals [11], steel making and metal refining [12], slag foaming [13], removal of non-metallic inclusions [14], and pore formation in solids [15], etc.

Recently, controlling microscale bubbles has been found to play a very important role in micro- and nano-sciences and technologies, such as ink-jet printing [16], pump and valves [17–20], ultrasound diagnostics [21], drug delivery and gene transfection [22], etc. The inkjet printer industry [16] is the earliest use of microbubbles to create a jet of fluid for printing. In a thermal inkjet, a short duration electric pulse around several microseconds is applied to a resistive heater to generate a high heat flux. The ink thus becomes highly superheated. This leads to sudden formation of a vapor bubble, which generates a pressure impulse to push the growth of the bubble. The rapid growth of the bubble then ejects a small drop of ink out of a nozzle located near the heater. Once the bubble collapses, the nozzle is refilled and ready for the next pulse. The process of thermal inkjet printing thus is divided into three stages, bubble nucleation and growth, bubble collapse, and droplet ejection.

Microbubbles have been also employed in microfabricated devices such as pumps, valves and actuators [17–20]. The absence of extraneous moving components in designs renders them highly practical for future microscale applications. The use of microbubbles in microfluidic devices has been demonstrated by Prosperetti and coworkers [17], who generated vapor bubbles by electrically heating a filtered saturated solution of table salt in water. In view of the difference in capillary pressures at different locations determined by the designed shape of the microchannel, the periodic generation and collapse of a single microbubble result in a pumping effect. Lin and Pisano [18] also demonstrated bubble-powered microactuators by using the pressure of a growing and collapsing microbubble to mechanically push and then release microcantilever plates. They further used the rapid expansion of a microbubble to provide a transient force to switch valves in a microdevice [19]. Electrolysis is another way to generate gas bubbles acting as transient pistons and valves [23]. Displaced bubble can also block the path of a light beam, creating an optical switch [24].

Using tiny bubbles to perform logical operations could lead to smart lab-on-a-chip devices for drug discovery and chemical and biological analyses. More recently, Prakash and Gershenfeld [25] have designed complex intersecting channels that direct tiny

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Nomenclature

Bo	Bond number, defined in Eq. (5)
h_B	liquid depth, as illustrated in Fig. 1
L	microbubble radius at $\pi/2$
p	pressure
$p_{\sigma 0}$	capillary pressure at microbubble apex, defined in Eq. (3)
r	cylindrical coordinate
R_0	initial or critical radius
$R(0)$	radius of curvature at microbubble apex
R_1, R_2	radii of principal curvature, $1/R_1 = d\phi/ds$, $1/R_2 = \sin\phi/r$
s	arc length measured from axisymmetric axis
z	cylindrical coordinate, as illustrated in Fig. 1
z_B	bubble height, as illustrated in Fig. 1

Greek letter

σ	surface tension
ϕ	inclination angle, as illustrated in Fig. 1

Subscripts

a	atmosphere
B	base
g	gas
I	inflection point
ℓ	liquid
N	neck
ou	outer limit
v	vapor
0	initial state

Superscripts

$\sim$	dimensional quantity
$*$	dimensionless quantity

bubbles to various locations on the microfluidics chip. Electronic devices control the frequency with which bubbles enter the microfluidic chip. Several types of intersections act as different types of logic gates, like those on computer chips. By controlling the timing of the bubbles and the types of logic gates used, researchers can design circuits on microfluidic chips for different tasks including computation, chemical and biological analyses. The idea is eventually to enclose biological samples inside the bubbles. Then the bubble logic circuits will make it possible to test hundreds of reagents on a DNA sample, for example, or make all possible combinations of different chemicals to discover possible drugs.

Microbubbles have also become increasingly popular as contrast enhancers in ultrasound diagnostics [21]. A solution of microbubbles is injected into the bloodstream. The bubbles scatter ultrasound more efficiently than tissue or blood, and thus permit efficient flow visualization. For strong ultrasound, the bubbles also emit sound in higher harmonics. Those harmonics allow for better contrast to tissue, which scatters sound at mainly the fundamental frequency. In other work, a new trend in bubble medicine is to use the same kind of microbubbles for therapy, in which the bubbles can act as vectors for directed drug delivery and gene transfection into living cells [22]. The permeability of cell walls for large molecules such as drugs and genes is dramatically increased in the presence of ultrasound and microbubbles.

Evidently, finding parameters to control the state and growth of a microbubble is of practical and academic importance. The growth of a bubble, however, exhibits rather complicated patterns. Chesters [26] is the first to propose that the observed growths and departure of a bubble at an orifice can be categorized into modes A and B. Mode A is referred to the growth of a bubble having good wettability and the base attached to the orifice edge. The bubble growth for mode B is for poor wettability resulting in the base to spread over the plate surrounding the orifice. In both models, the bubble expands and then shrinks. As the concave radius of the neck vanishes, the bubble departs from the wall. The transition between two modes was ruled by actual contact angle, determined from the bubble profile by solving Young-Laplace equation [27]. If the actual contact angle is less than the equilibrium contact angle, mode B took place. Proposing departure as the maximum volume is reached, bubble sizes at detachment in modes A and B and forces involved during the bubble growth were estimated.

Fundamental and simple analyses can often give good insight into understanding of complicated growths of a bubble. Gerlach et al. [28] numerically solved Young-Laplace equation to predict static formation of air bubbles emanating from an orifice. Instead of applying the theory of contact angle hysteresis, bubble formations in modes A and B were predicted by choosing different boundary conditions, for example, a given radius of the bubble base, which is allowed to spread over the surrounding plate. The results showed that the predicted bubble shapes for both modes agreed well with measurements by using a shadow imaging technique. Mode A was found to depend on the ratio between orifice radius and Laplace constant. Transition from mode A to B occurred if the instantaneous contact angle fell below the equilibrium contact angle. The bubble formation for mode B contracted towards the orifice before departure. The sequences of departure thus include transitions from mode A to B and then B to A.

The general solutions for the microbubble shape can be adopted from analytical approximate solutions of small sessile and pendant drops subject to a small Bond number, as presented by Chesters [27] and O'Brien [29], respectively. Small Bond number implies hydrostatic pressure to be much less than surface tension. Since the first-order perturbation for hydrostatic pressure results in singularity at inclination angle π, the solution provided by Chesters [27] was improved by O'Brien [29] by applying the formal singular perturbation method to obtain solutions of a pendant drop in the outer, lower and upper interior-layers, neck and top boundary layer regions. The bubble shape in region I is described by an outer limit solution where capillary and net pressures are the dominant forces perturbed by hydrostatic pressure, and upper-interior-layer solution in the absence of hydrostatic pressure. Bubble shape in region II contains the lower-interior-layer solution where hydrostatic pressure is neglected, and a solution in the neck region where capillary pressure is the only force accounted for. Since solution in the neck region exists singularity as the inclination angle approaches zero near the base, bubble shape in region III includes solutions of the neck region, and a boundary layer where net and capillary pressures are the dominant forces. O'Brien [29], however, did not extensively study the effects of dimensionless parameters and boundary condition at the base on the bubble shape. Determination of the radius at inclination angle of $\pi/2$ chosen as a length scale was not presented. Detailed and analytical investigations of bubble growth, entrapment, or necking and departure affected by different working parameters have not been generally presented. This becomes the objective of this work.