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Finite element based heatline approach to study mixed convection in a porous square cavity with various wall thermal boundary conditions

Tanmay Basak [a], P.V. Krishna Pradeep [a], S. Roy [b], I. Pop [c,*]

[a] Department of Chemical Engineering, Indian Institute of Technology Madras, Chennai 600036, India
[b] Department of Mathematics, Indian Institute of Technology Madras, Chennai 600036, India
[c] Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania

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ABSTRACT

A penalty finite element method based simulation is performed to analyze the influence of various walls thermal boundary conditions on mixed convection lid driven flows in a square cavity filled with porous medium. The relevant parameters in the present study are Darcy number ($Da = 10^{-5} - 10^{-3}$), Grashof number ($Gr = 10^3 - 10^5$), Prandtl number ($Pr = 0.7–7.2$), and Reynolds number ($Re = 1–10^2$). Heatline approach of visualizing heat flow is implemented to gain a complete understanding of complex heat flow patterns. Patterns of heatlines and streamlines are qualitatively similar near the core for convection dominant flow for $Da = 10^{-3}$. Symmetric distribution in heatlines, similar to streamlines is observed irrespective of *Da* at higher *Gr* in natural convection dominant regime corresponding to smaller values of *Re*. A single circulation cell in heatlines, similar to streamlines is observed at $Da = 10^{-3}$ for forced convection dominance and heatlines are found to emanate from a large portion on the bottom wall illustrating enhanced heat flow for $Re = 100$. Multiple circulation cells in heatlines are observed at higher *Da* and *Gr* for $Pr = 0.7$ and 7.2. The heat transfer rates along the walls are illustrated by the local Nusselt number distribution based on gradients of heatfunctions. Wavy distribution in heat transfer rates is observed with $Da \geqslant 10^{-4}$ for non-uniformly heated walls primarily in natural convection dominant regime. In general, exponential variation of average Nusselt numbers with Grashof number is found except the cases where the side walls are linearly heated. Overall, heatlines are found to be a powerful tool to analyze heat transport within the cavity and also a suitable guideline on explaining the Nusselt number variations.

1. Introduction

The study of fluid flow and heat transfer induced by the combined effects of the mechanically driven lid and buoyancy force within closed enclosures filled with fluid saturated porous medium is of great interest due to high surface-area density. Various applications on convection in porous medium involve use of metal foams for enhanced cooling in electronic equipment, foam filled heat exchangers, open-cell metal foams, use of fibrous materials in thermal insulation of buildings, solar energy collectors, crystal growing, post-accidental heat removal in nuclear reactors to name just a few of them [1–5]. In mixed convection flows, the forced convection and the free convection effects are of comparable magnitudes. In case of lid-driven cavity flows, the thermal non-homogeneity gives rise to buoyancy force which in turn impacts upon the coupled fields of velocity and temperature in the cavity. The governing non-dimensional parameters for mixed convection in a cavity filled with fluid saturated porous medium are Darcy number (*Da*), Grashof number (*Gr*), Reynolds number (*Re*) and Prandtl number (*Pr*). Note that, *Gr* and *Re* represent the strength of the natural and forced convection flow effects, respectively. A comprehensive review on the fundamentals of the convective flow in porous media can also be found in the literature [6–11].

Numerical and experimental studies on mixed convection in porous media have received significant attention of investigators due to various engineering applications [12,13]. The numerical heat transfer characteristics of non-Darcy mixed convection flow over a horizontal flat plate with porous medium was studied by Chen [12]. Darcy–Brinkman–Forchheimer equation to model the motion of fluid through porous medium has been used in this study. Laminar transport processes in a lid driven porous square cavity saturated with water was investigated by Al-Amiri [13]. A few earlier investigations also involve detailed analysis of mixed convection flow over vertical surface in porous medium [14–18]. Oztop [14] investigated numerical heat transfer and fluid flow in a porous lid driven cavity with isothermal moving top wall. The effects of the flow governing parameters on the characteristics of the flow and thermal fields on mixed convective heat transfer in

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* Corresponding author.

E-mail addresses: tanmay@iitm.ac.in (T. Basak), pandithapradeep@gmail.com (P.V. Krishna Pradeep), sjroy@iitm.ac.in (S. Roy), popm.ioan@yahoo.co.uk (I. Pop).

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Nomenclature

Da	Darcy number
g	acceleration due to gravity, m s^{-2}
k	thermal conductivity, W m^{-1} K^{-1}
K	permeability, m^2
L	height of the square cavity, m
Nu	local Nusselt number
p	pressure, Pa
P	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
Re	Reynolds number
Gr	Grashof number
T	temperature, K
T_h	temperature of hot bottom wall, K
T_c	temperature of cold wall, K
u	x component of velocity
U	x component of dimensionless velocity
v	y component of velocity
V	y component of dimensionless velocity
X	dimensionless distance along x coordinate
Y	dimensionless distance along y coordinate

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β	volume expansion coefficient, K^{-1}
γ	penalty parameter
θ	dimensionless temperature
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
Φ	basis functions
ψ	streamfunction
Π	heatfunction

Subscripts

b	bottom wall
k	node number
s	side wall

rectangular enclosures driven by a continuously moving horizontal plate was studied by Waheed [15]. Steady mixed convection flow in a vented enclosure with an isothermal vertical wall and filled with a fluid-saturated porous medium is investigated by Mahmud and Pop [16]. Duwairi et al. [17] analyzed the effects of oscillating plate temperature on transient mixed convection heat transfer from a porous vertical surface embedded in a saturated porous medium with internal heat generation or absorption. Jue [18] investigated mixed convection flow caused by a torsionally oscillatory lid with thermal stable stratification in an enclosure filled with porous medium using semi-implicit projection finite-element method.

Till date, most of the numerical investigations on lid driven enclosures filled with fluid saturated porous medium are limited to analysis based on streamlines and isotherms and the detailed analysis of heat flow was not well understood. The present work is carried out on visualization of heat flow to analyze optimal thermal mixing and temperature distributions within porous square cavities filled with different fluids in presence of a moving top wall. Current work attempts for the first time to analyze heat transfer, correlations and energy distributions using heatline approach for mixed convection in a cavity filled with porous medium.

The heatline is found to be the best numerical tool to visualize the heat transport in two dimensional convective transport process. Heatlines refer to trajectories of total heat transport involving conductive as well as convective heat flux. In convective heat transport, the energy flow within various regimes can be best visualized by heatlines as the isotherms are unable to give guideline for energy flows. Heatlines are found via solving the governing equations of heatfunctions and each heatline contour corresponds to constant heatfunction. It may be noted that the derivative of heatfunctions are defined as a combination of conductive and convective heat flux and various directional derivatives of heatfunctions are obtained from energy balance equations. Proper dimensionless forms of heatfunctions are closely related to Nusselt numbers. The concept of heatline was first introduced by Kimura and Bejan [19,20]. Over the years, heatlines have been employed as effective tool to describe various physical phenomenon [21–24].

A few earlier studies on heatlines were carried out for thermal convection analysis and in analyzing heat flow in electroconductive melts [25,26]. Zhao et al. [27,28] studied natural convection in a porous enclosure with heat and solute sources and illustrated the flow characteristics via streamlines, heatlines, isotherms and masslines. Heatline patterns for the fluid with temperature dependent viscosity in a porous square cavity was reported by Hooman and Gurgenci [29]. Heat flow visualization in a complicated cavity has been studied by Dalal and Das [30] using the heatline concept. Effects of wall-located heat barrier on conjugate conduction/natural-convection heat transfer and fluid flow in enclosures have been studied using heatlines by Hakyemez et al. [31]. The concept of masslines has been introduced, analogous to heatlines to visualize mass transfer within the cavity [32–37]. However, a detailed analysis of heat flow using heatline concept for lid driven flows in square enclosures filled with porous medium is yet to appear in the literature.

The aim of the current study is to analyze the heat flow due to mixed convection in a square cavity filled with a fluid saturated porous medium for various thermal boundary conditions as a first attempt. The main objective of the present study is to examine the extent of thermal mixing and heat transfer within the porous cavity in the presence of a moving top wall. A square cavity with four different thermal boundary conditions has been considered in the current study. A penalty finite element approach using the Galerkin method is applied to solve the non-linear coupled equations for flow and temperature fields. The Galerkin method is further employed to solve the Poisson equation for streamfunctions and heatfunctions. Finite discontinuity exists at the junction of hot and cold walls leading to mathematical singularity. Solution of heatfunction for such type of situation demands implementation of exact boundary conditions. Each case is studied for a range of parameters: Darcy number ($Da = 10^{-5} - 10^{-3}$), Grashof number ($Gr = 10^3 - 10^5$), Prandtl number ($Pr = 0.7–7.2$), and Reynolds number ($Re = 1 - 100$). Numerical results are obtained for velocity and thermal fields within the cavity and are displayed using streamlines, isotherms and heatlines.

2. Mathematical formulation and simulation

The physical domain consists of a square cavity with the physical dimensions as shown in Fig. 1. The top wall is assumed to move with a uniform velocity U_o. Four cases in the present study are considered as follows: case 1: bottom wall is uniformly heated where the side walls are maintained cold, case 2: bottom wall is