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Heat transfer analysis for a hydromagnetic viscous fluid over a non-linear shrinking sheet

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ABSTRACT

The boundary layer flow and heat transfer analysis of electrically conducting viscous fluid over a nonlinearly shrinking sheet is investigated. A similarity transformation is used to reduce the governing equations to a set of nonlinear ordinary differential equations. The system of equations is solved numerically employing an implicit finite difference scheme known as Keller-box method. It is found that dual solutions exist for this particular problem. The numerical results for the velocity, temperature, wall skin friction coefficient and local rate of heat transfer through the surface for various values of physical parameters both in case of stretching and shrinking sheet are analyzed and discussed for both the solutions. Present results in the hydrodynamic case ($M = 0$) are compared with existing numerical results in case of stretching flow and found in good agreement.

1. Introduction

The flow and heat transfer analysis in the boundary layer on a continuously moving or stretching surface has important applications in many manufacturing processes and polymer industry, for examples, a continuous stretching of plastic films, artificial fibers, metal spinning, metal extrusion, continuos casting, glass blowing and many more. The pioneering work on the continuously stretching sheet was first initiated by Sakiadis [1]. The problem in [1] is extended to discuss the various aspects of flow and heat transfer characteristics by many researchers [2–10]. The linear velocity of the surface is taken in all above problems. In 2001, Vajravelu [11] studied the numerical solution of flow and heat transfer in a viscous fluid over a nonlinearly stretching sheet. Later, Cortell [12] discussed the viscous flow and heat transfer with constant surface temperature (CST) and prescribed surface temperature (PST) cases numerically. Recently, Abbas and Hayat [13] extended the work of [12] by considering the electrically conducting fluid and radiation effects in a porous medium and obtained purely analytic solution using homotopy analysis method (HAM). Very recently, Bataller [14] discussed the similarity solutions for flow and heat transfer of a quiescent fluid over a non-linearly stretching sheet. He assumed a variable wall temperature and obtained a numerical solution.

The problem in the reverse case i.e., very little is known about the shrinking sheet where the velocity on the boundary is towards the origin or a fixed point, and the unsteady shrinking film solution was first investigated by Wang [15]. Again, Miklavcic and Wang [16] studied the viscous hydrodynamic flow over a shrinking sheet for both two-dimensional and axisymmetric flows. It is also noted that the mass suction at the wall is required generally to maintain (or smooth) the flow over a shrinking sheet. They discussed the proof of existence and (non) uniqueness of both exact numerical and closed form solutions. The analysis of [16] was also extended in various directions for different fluids by many researchers [17–22]. Recently, Fang [23] investigated the boundary layer flow over a shrinking sheet with surface moving with power-law velocity. A theoretical analysis is carried out for different values of power-index of the surface velocity using exact and numerical solutions.

The aim of the present paper is two fold: first to incorporate the effects of applied magnetic field for an electrically conducting fluid, secondly to carry out the heat transfer analysis. The system of nonlinear coupled ordinary differential equations is solved numerically using an implicit finite difference scheme known as Keller-box method.

2. Mathematical formulation

Consider a two-dimensional flow of an incompressible viscous fluid past a porous shrinking sheet at $y = 0$. The stretching/shrinking velocity of the sheet is $U_w(x) = \epsilon c x^n$, where $\epsilon = 1, \ -1$ is

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respectively for stretching and shrinking sheet and the wall mass suction velocity is $v_w = v_w(x)$. The x-axis is taken along the stretching/shrinking sheet and the y-axis perpendicular to it into the fluid. The fluid is electrically conducting and the magnetic field $B(x)$ is assumed to be applied in the y-direction. The magnetic Reynolds number is taken to be small so that the induced magnetic field can be neglected. The temperature of the surface maintained at a constant temperature $T_w = T_0$ and far away from the sheet temperature is T_∞, where $T_w > T_\infty$. Under boundary layer approximation, the continuity, momentum, and energy equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}u, \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are the velocity components in the x- and y-directions, respectively, ρ is the fluid density, μ is the dynamic viscosity, ν is the kinematic viscosity, σ is the electrical conductivity of the fluid, T is the temperature, c_p is the specific heat at constant pressure and k is the thermal diffusivity. In Eq. (2), the external electric field and the polarization effects are neglected and [24]

$$B(x)=B_0x^{(m-1)/2}.$$

The boundary conditions for the problem are

$$u=U_w(x)=\epsilon c x^m,\quad v=v_w(x)T=T_0 \text{ at } y=0, \tag{4}$$

$$u\to 0,\quad T\to T_\infty \text{ as } y\to\infty \tag{5}$$

where m is a power index and c is a constant rate stretching/shrinking which has a dimension of $(\text{time})^{-1}$.

We are interested in obtaining a similarity solution of the form

$$\eta=y\sqrt{\frac{c(m+1)}{2\nu}}x^{\frac{m-1}{2}},\quad u=cx^mf'(\eta),$$
$$v=-\sqrt{\frac{c\nu(m+1)}{2}}x^{\frac{m-1}{2}}\left[f(\eta)+\frac{m-1}{m+1}\eta f'(\eta)\right],\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

and the wall mass transfer velocity becomes [23]

$$v_w(x)=-\sqrt{\frac{c\nu(m+1)}{2}}x^{\frac{m-1}{2}}f(0)\propto x^{\frac{m-1}{2}}. \tag{7}$$

Using Eq. (6), the continuity Eq. (1) is identically satisfied and Eqs. (2) and (3) yield

$$f'''+ff''-\beta f'^2-M^2f'=0, \tag{8}$$

$$\theta''+\mathrm{Pr}f\theta'+\mathrm{Pr}Ecf''^2=0 \tag{9}$$

and the boundary conditions become

$$f(0)=s,\quad f'(0)=\epsilon,\quad f'(\infty)=0, \tag{10}$$

$$\theta(0)=1,\quad \theta(\infty)=0, \tag{11}$$

where s is the wall mass transfer at the sheet and

$$\beta=\frac{2m}{m+1},\quad M^2=\frac{2\sigma B_0^2}{c\rho(m+1)},\quad \mathrm{Pr}=\frac{\mu c_p}{k},\quad Ec=\frac{U_w^2(x)}{c_p(T_w-T_\infty)}.$$

It is noted that in the special case when $M = 0$ and $m = 1$, the Eq. (8) is reduced to the problem discussed by Miklavcic and Wang [16]. The problem discussed by Fang [23] can be retrieved when $M = 0$.

When $m = 1$, Eq. (8) takes the form

$$f'''+ff''-f'^2-M^2f'=0. \tag{12}$$

The exact solution of the above equation can be written as

$$f=s+\frac{\epsilon}{b}\left(1-e^{-b\eta}\right). \tag{13}$$

where

$$b_{1,2}=\frac{1}{2}\left(s\pm\sqrt{4\epsilon+4M^2+s^2}\right).$$

For $s = 0$ the solution of the MHD flow in the case of rigid plate was discussed by Pavlov [26]. In that case a unique solution is obtained for $b=\sqrt{1+M^2}$. However for a porous plate dual exact solutions also exist.

The shear stress at the wall is given by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)\bigg|_{y=0}. \tag{14}$$

The above equation in dimensionless form becomes

$$\tau_w=c\mu\sqrt{\frac{c(n+1)}{2\nu}}x^{\frac{3n-1}{2}}f''(0) \tag{15}$$

and the local rate of heat transfer of the surface is

$$-k\left(\frac{dT}{dy}\right)_{y=0}=-k(T_0-T_\infty)\theta'(0)x^{\frac{n-1}{2}}\sqrt{\frac{c(n+1)}{2\nu}}. \tag{16}$$

3. Results and discussion

The system of nonlinear ordinary differential Eqs. (8) and (9) with boundary conditions (10) and (11) has been solved numerically using an implicit finite difference scheme known as the Keller-box method and described in the book by Cebeci and Bradshaw [25]. This method is described in the following four steps:

(i) Given nonlinear system of boundary value problems (8)–(11) is reduced to a first order system.
(ii) Obtained system of first order is reduced to system of algebraic equations by using central difference formula.
(iii) Linearize the resulting algebraic equation by using Newton's method and arrange then in matrix–vector form.
(iv) and then linear system of algebraic equations is solved by using box-tridiagonal-elimination method.

It is worth mentioning here that the step size $\Delta\eta$and the boundary layer thickness is chosen according to the values of parameters. The iteration process is repeated until the accuracy up to 10^{-10} is achieved.

The effects of various parameters for example, the power-index m of the surface velocity, magnetic field or Hartman number M, mass suction parameter s, and the Prandtl number Pr on the velocity $f(\eta)$, the shear stress at the wall $f''(0)$, temperature field θ and the local rate of heat transfer at the surface or the local Nusselt number $-\theta'(0)$ are shown in Figs. 1–11. The numerical values of the skin friction (or shear stress) at the wall $f''(0)$ and the local rate of heat transfer $-\theta'(0)$ for different values of parameters of interest are also tabulated in Tables 1–4.

In Fig. 1 the velocity $f(\eta)$ is plotted both for numerical (solid line) and exact (filled circles) solutions in case of linear stretching sheet with $m = 1$, $M = 0.1$ and $s = 3.5$ are fixed. Fig. 2 presents the plots of $f(\eta)$ showing a comparison between numerical (solid line) and exact (filled circles) solutions in case of linear shrinking sheet for $m = 1$, $M = 0.1$ and $s = 3.5$. It is noted that dual solutions exist for flow due to shrinking sheet in both the cases of numerical and exact solutions for $M = 0.1$ and $s = 3.5$, and we call them as first and second solution. From these two figures, it is further found that both exact and numerical solution are identically same. Figs. 3 and