International Journal of Heat and Mass Transfer 54 (2011) 2116–2125

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# The onset of Brinkman ferroconvection using a thermal non-equilibrium model

I.S. Shivakumara [a,b], Jinho Lee [a,*], M. Ravisha [c], R. Gangadhara Reddy [d]

[a] School of Mechanical Engineering, Yonsei University, Seoul 120-749, South Korea
[b] UGC-Centre for Advanced Studies in Fluid Mechanics, Department of Mathematics, Bangalore University, Bangalore 560 001, India
[c] Smt. Rukmini Shedthi Memorial National Government First Grade College, Department of Mathematics, Barkur 576210, Udupi District, India
[d] Department of Mathematics, AMC Engineering College, Bangalore 560 083, India

ARTICLE INFO

*Article history:*
Received 30 June 2010
Received in revised form 10 November 2010
Accepted 8 December 2010
Available online 13 January 2011

*Keywords:*
Brinkman model
Ferrofluid
Porous medium
Thermal non-equilibrium

ABSTRACT

The onset of ferroconvection in a horizontal layer of ferrofluid saturated Brinkman porous medium heated uniformly from below in the presence of a uniform vertical magnetic field is investigated when the solid and fluid phases of the porous medium are in local thermal non-equilibrium (LTNE) using linear stability theory. The modified Brinkman–Forchheimer-extended Darcy equation is employed to describe the flow in the porous medium and a two-field model for energy equation each representing the solid and fluid phases separately is used. It is established that the principle of exchange of stability is valid. The authenticity of LTNE model over LTE model and also the ferromagnetic effects on the stability of the system are discussed in detail. The system is found to be more stable when the magnetic forces alone are present. Asymptotic analysis for both small and large values of scaled inter-phase heat transfer coefficient $H_t$ is presented and the results are found to be in good agreement with those obtained from the exact formula. The established results in the literature have been reproduced as particular cases from the present study.



## 1. Introduction

Ferrofluids are surfactant- coated magnetic nanoparticles in the size range of 2–10 nm dispersed in liquid carriers like water, hydrocarbon (mineral oil or kerosene) or fluorocarbon. They have an ability to respond to an external magnetic field and being unique materials exhibit both liquid and magnetic properties. Hence they are exploited in a variety of applications and the details can be found in the books by Rosensweig [1] and Berkovsky et al. [2]. The magnetization of ferrofluids depends on the magnetic field, temperature, and density. Hence, any variations of these quantities induce change of body force distribution in the fluid and eventually give rise to convection in ferrofluids in the presence of a gradient of magnetic field. Of late, researchers are looking for new technologies to improve the operation of existing oil-cooled electromagnetic equipment. One of the approaches suggested in the literature is to replace the oil in such devices with oil-based ferrofluids, which can take advantage of the pre-existing leakage magnetic fields to enhance heat transfer processes. Hence the study of thermal convection in magnetized ferrofluids has triggered lot of research interest on this topic considering the prospect of heat transfer applications in electronics, engines, and micro and nanoelectromechanical systems.

There have been numerous studies carried out on thermal convection in a ferrofluid layer heated from below called ferroconvection analogous to Rayleigh–Benard convection in ordinary viscous fluids. In his review article, Odenbach [3] has focused on recent developments in the field of rheological investigations of ferrofluids and their importance for the general treatment of ferrofluids. By considering variety of velocity and temperature boundary conditions, a detailed study on the onset of ferroconvection in an initially quiescent ferrofluid layer has been made by Nanjundappa and Shivakumara [4]. Matura and Lucke [5] have investigated the influence of time periodic and spatially homogeneous magnetic field on the linear and nonlinear stability of a ferrofluid layer heated from above and below. Thermal convection of ferrofluids in the presence of a uniform vertical magnetic field with the boundary temperatures modulated sinusoidally about some reference values has been studied by Singh and Bajaj [6] with the idea of understanding control of ferroconvection.

The problem of ferroconvection in a ferrofluid saturated porous medium has also attracted considerable attention in the literature owing to its importance in controlled emplacement of liquids or treatment of chemicals, and emplacement of geophysically imageble liquids into particular zones for subsequent imaging etc. Rosensweig et al. [7] have studied experimentally the penetration of ferrofluids in the Hele-Shaw cell. The stability of the magnetic fluid penetration through a porous medium in a uniform magnetic field oblique to the interface is studied by Zahn and Rosensweig [8]. The thermal convection of ferrofluid saturating a porous medium in the

* Corresponding author.
*E-mail addresses:* shivakumarais@gmail.com (I.S. Shivakumara), jinholee@yonsei.ac.kr (J. Lee).

0017-9310/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijheatmasstransfer.2010.12.016

**Nomenclature**

| | |
|---|---|
| $a = \sqrt{\ell^2 + m^2}$ | overall horizontal wave number |
| $B$ | magnetic induction |
| $c$ | specific heat |
| $c_F$ | dimensionless form drag constant |
| $d$ | thickness of the porous layer |
| $D = d/dz$ | differential operator |
| $Da = k/d^2$ | Darcy number |
| $\vec{g}$ | acceleration due to gravity |
| $h$ | heat transfer coefficient |
| $H$ | magnetic field intensity |
| $H_0$ | imposed uniform vertical magnetic field |
| $H_t = hd^2/\varepsilon k_{tf}$ | scaled inter-phase heat transfer coefficient |
| $\hat{k}$ | unit vector in $z$-direction |
| $k$ | permeability of the porous medium |
| $k_t$ | thermal conductivity |
| $K = -(\partial M/\partial T)_{H_0,T_0}$ | pyromagnetic co-efficient |
| $\ell, m$ | wave numbers in the $x$ and $y$ directions |
| $M$ | magnetization |
| $M_0 = M(H_0, T_a)$ | constant mean value of magnetization |
| $M_1 = \mu_0 K^2 \beta/(1+\chi)\alpha_t \rho_0 g$ | magnetic number |
| $M_3 = (1 + M_0/H_0)/(1+\chi)$ | non-linearity of magnetization parameter |
| $p$ | pressure |
| $\vec{q} = (u, v, w)$ | velocity vector |
| $R = \rho_0 \alpha_t g \beta d^2/\varepsilon \mu_f \kappa_f$ | Rayleigh number |
| $R_m = RM_1$ | magnetic Rayleigh number |
| $t$ | time |
| $T$ | temperature |
| $T_l$ | temperature of the lower boundary |
| $T_u$ | temperature of the upper boundary |
| $T_a = (T_l + T_u)/2$ | reference temperature |
| $W$ | amplitude of vertical component of perturbed velocity |
| $(x, y, z)$ | Cartesian co-ordinates |

*Greek symbols*

| | |
|---|---|
| $\alpha_t$ | thermal expansion coefficient |
| $\beta = \Delta T/d$ | temperature gradient |
| $\chi = (\partial M/\partial H)_{H_0,T_0}$ | magnetic susceptibility |
| $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ | Laplacian operator |
| $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ | horizontal Laplacian operator |
| $\varepsilon$ | porosity of the porous medium |
| $\kappa_f = k_{tf}/(\rho_0 c)_f$ | thermal diffusivity of the fluid |
| $\Lambda = \tilde{\mu}_f/\mu_f$ | ratio of viscosities, |
| $\mu_f$ | dynamic viscosity |
| $\tilde{\mu}_f$ | effective viscosity, |
| $\mu_0$ | free space magnetic permeability of vacuum |
| $\varphi$ | magnetic potential |
| $\Phi$ | amplitude of perturbed magnetic potential |
| $\gamma = \varepsilon k_{tf}/(1-\varepsilon)k_{ts}$ | porosity modified conductivity ratio. |
| $\rho_f$ | fluid density |
| $\rho_0$ | reference density at $T_a$ |
| $\Theta$ | amplitude of temperature |

*Subscripts*

| | |
|---|---|
| $b$ | basic state |
| $f$ | fluid |
| $s$ | solid |

presence of a vertical magnetic field is studied by Vaidyanathan et al. [9] by employing the Brinkman equation. Qin and Chadam [10] have carried out the non-linear stability analysis of ferroconvection in a porous layer by including the inertial effects to accommodate high velocity. The laboratory–scale experimental results of the behaviour of ferrofluids in porous media consisting of sands and sediments are presented by Borglin et al. [11]. Sunil and Amit Mahajan [12] have used generalized energy method to study nonlinear convection in a magnetized ferrofluid saturated porous layer heated uniformly from below. Recently, Shivakumara et al. [13,14] have investigated theoretically the onset of convection in a layer of ferrofluid saturated porous medium for various types of velocity and temperature boundary conditions.

It has been realized that the assumption of local thermal equilibrium (LTE) is inadequate for proper understanding of the heat transfer problems in many practical applications involving hyperporous materials and also media in which there is a large temperature difference between the fluid and solid phases. In such circumstances, the local thermal non-equilibrium (LTNE) effects are to be taken into consideration. Therefore, the recent trend in the study of thermal convective instability problems in porous media is to account for LTNE effects by considering a two-field model for energy equation each representing the fluid and solid phases separately. Several investigations have been carried out in the recent past to know LTNE effects on forced and free convection in porous media involving ordinary viscous fluids. The LTNE effects on forced and free convection flows in a porous medium have been covered exhaustively in the book by Nield and Bejan [15]. Using LTNE model, Banu and Rees [16] have discussed the onset of convection in a Darcy porous medium, while Malashetty et al. [17] have analyzed the problem by considering the Brinkman model for stress-free boundaries. Recently, Shivakumara et al. [18,19] have used LTNE model and analyzed the effects of temperature dependent viscosity and quadratic density as well as boundary effects on the onset of free convection in a layer of porous medium.

Although copious literature is available on ferroconvection in a ferrofluid saturated porous medium, the studies are limited to LTE model and no attention has been given to understand LTNE effects on the criterion for the onset of ferroconvection in spite of its relevance and importance in many heat transfer problems as noted above. Recently, Lee et al.[20] have investigated the effect of LTNE on convective instability in a layer of ferromagnetic fluid saturated Darcy porous medium heated from below in the presence of a uniform vertical magnetic field. However, it is realized that the Darcy model is applicable only under special circumstances and a generalized model for the accurate prediction of convection in a porous media must include Brinkman viscous term and Forchheimer inertia term. During the last decade, there has been a great upsurge of interest in determining the effects of extensions to Darcy's law since many practical applications involve porous media for which Darcy's law is inadequate.

The intent of the present study is, therefore, to investigate how the onset criterion for ferroconvection in a horizontal ferromagnetic fluid saturated sparsely packed porous layer heated from below in the presence of an imposed spatially uniform magnetostatic field in the vertical direction is affected when the solid and fluid phases of the porous medium are in LTNE. The flow in the porous medium is described by the modified Brinkman–Forchheimer-extended Darcy equation. The bounding surfaces of the porous layer are considered to be stress free, ferromagnetic, and perfect thermal conductors. The criterion for the onset of convection is analyzed analytically and the existing results are obtained as particular cases from the present study. Since the basic state considered is quiescent, it is observed that the inertia has no influence on the stability criteria.