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Volumetric maximization of coolant usage in closed self-driven circuits

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ABSTRACT

The present study numerically maximizes the heat transferred in a rectangular loop. The loop or circuit is composed of two superimposed rectangles of different sizes with coolant filling the vacant space between them. Buoyancy forces promote coolant motion, since a constant temperature difference is maintained between the two outer vertical walls of the circuit. The results are presented in terms of two quantities: the heat transfer rate per unit of length and per unit of coolant volume, for several values of the Rayleigh number. The numerical solution is obtained by applying the finite elements method to the two-dimensional numerical domain, here represented by the coolant only. The numerical results show that the relative size of the inner and outer rectangles composing the circuit are important in terms of thermal performance and that the optimal gap size between the inner and outer rectangles decreases as the Rayleigh is increased. The circuit aspect ratio (i.e., height/width) was also investigated revealing to have a positive effect on the overall thermal performance of the system if increased, while the eccentricity of the two rectangles composing the circuit presented an opposite effect. The numerical results were compared with scaling analysis showing good agreement.

1. Introduction

Natural convection in idealized enclosed spaces has been a topic of interest for many years, e.g., [1,2], attracting researchers who relied on such configurations to provide fundamental insights in real devices. Among the many configurations considered, possibly the most common so far is the one where a square or rectangular enclosure is differentially heated, e.g., [3,4]. Variations of this configuration have also been addressed in several occasions [5–10].

Thermal energy transfer in enclosed spaces has been also investigated while accounting for different heat transfer modes. Certainly, the purely convective heat transfer approach is the most prolifically found [1–4,11,12]. However, conjugated studies (i.e., conduction coupled with convection) have also been extensively reported [13–16]. Less often found is the study of fully coupled conduction, convection and radiation processes in enclosed spaces [17–20].

A more recent trend of studies has heavily focused on the optimization of numerous configurations, with constructal theory being one of the most predominantly used methods [21,22]. While the shape optimization of enclosed spaces (i.e., aspect ratio of differentially heated enclosures) represents early attempts to improve the performance of this configuration [2,23], recent studies based on constructal theory have shown that the allocation of heating and cooling within enclosed spaces can also significantly affect its performance, e.g., [24].

Another configuration derived from self-driven enclosed flows are the so-called closed circuits. These configurations can be seen as a differentially heated cavity with an obstruction at the center, which forces the flow to follow a narrow pathway between the obstruction and the external walls of the enclosure composing then a loop or circuit. The study of circuits has also interested the research community over the years [25–32].

Furthermore, it is well known that flow patterns inside differentially heated enclosures allow the appearance of large thermal and velocity gradients [3,4,22]. As a consequence, part of the available coolant, which here represents the heat carrier, is underutilized [21,22,33]. In this context, the present paper studies the performance of a differentially heated rectangular loop where its efficiency is measured by the heat transfer density of the coolant within [21,22] – this measure of efficiency has led to numerous optimized configurations, as shown by constructal theory [21,22,34–37]. The goal is to numerically maximize the amount of thermal energy transferred across the loop per unit of fluid volume (i.e., "volumetric maximization") [21,22], while accounting for different geometric factors and flow strengths, here represented by the Rayleigh number.

2. Foundations of the problem

Consider a two-dimensional enclosure of fixed area. Next, assume that an area with impermeable walls is inserted into the enclosure in a way that channels are created between each of the

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Nomenclature

A	area [m^2]
c_p	specific heat [J/kg K]
C	constants
D	channel breadth, Eq. (10) [m]
g	gravity [m/s^2]
H	enclosure height [m]
k	thermal conductivity [W/m K]
L	enclosure length [m]
$\overline{Nu}$	Average Nusselt number
P	pressure [Pa]
Pr	Prandtl number
q'	heat transfer rate [W/m]
q'''	heat transfer density [W/m^3]
r	center-to-center distance [m]
Ra	Rayleigh number
S	channel breadth [m]
T	temperature [K]
x, y	rectangular coordinates [m]
$\vec{V}$	velocity vector [m/s]

Subscripts

c	cold
h	horizontal or hot
i	internal
v	vertical

Superscript

$\sim$	dimensionless variables

Greek letters

α	thermal diffusivity [m^2/s]
β	expansion coefficient [K^{-1}]
λ	enclosure aspect ratio
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]

two parallel walls (i.e., loop; see Fig. 1). Finally, because the area of the outer cavity is fixed ($A = H \times L$), one can control the 'amount' of coolant inside the loop by varying the dimensions of the inner area ($A_i = H_i \times L_i$). Furthermore, assuming that the heated side of the enclosure is maintained at T_h and the cold wall at T_c (See Fig. 1), the heat transferred across the loop per unit of fluid volume [21,22] can be defined as

$$\tilde{q}''' = \frac{q'''}{k\Delta T/A} = \frac{\tilde{q}'}{(1-\tilde{A}_i)} \qquad (1)$$

where $\tilde{q}' = q'/k\Delta T = \int_0^{\tilde{H}} (\partial\tilde{T}/\partial\tilde{x})\mathrm{d}\tilde{y}$ is the dimensionless heat flow per unit length transferred from the heated wall to the fluid and $(1-\tilde{A}_i)$ is the dimensionless area of the coolant after inserting the inner enclosure-all non-dimensional variables are defined in Eq. (5). To foresee the existence of an optimum value of $\tilde{A}_i$, consider the configuration where $\tilde{A}_i = 0$. In this case and according to Eq. (1), $\tilde{q}'''$ is numerically equal to $\tilde{q}'$. Even though fluid can flow easily in the cavity with no obstacle, a centrally located stagnant fluid pocket that contributes poorly to heat transfer exists [21,22]. However, for the case where $\tilde{A}_i > 0$, the amount of coolant available is reduced, which is potentially beneficial to $\tilde{q}'''$ as shown in Eq. (1).

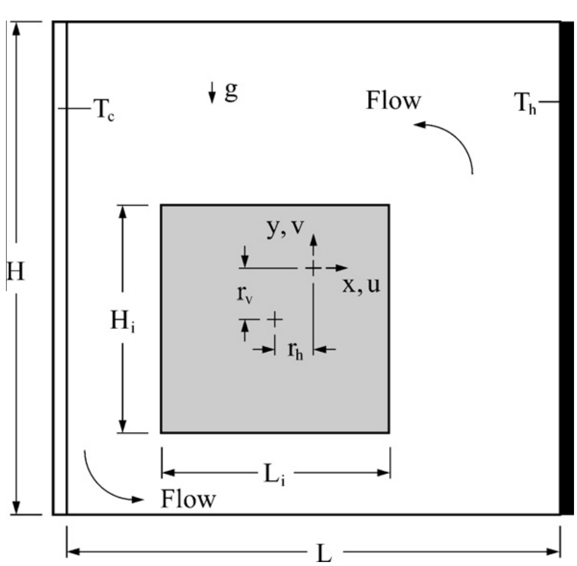

Fig. 1. Numerical domain.

3. Numerical method

The numerical method relied on the finite element approach to solve the conservation equations of mass, momentum and energy, where the fluid was considered to be incompressible with constant properties. The buoyancy term, responsible for the fluid motion, was added through the Boussinesq approach in the vertical component of the momentum equation. The conservation equations, which were normalized using the same variables considered by de Vahl Davis [3] and de Vahl Davis and Jones [4], are shown below:

$$\nabla \cdot \vec{V}^* = 0 \qquad (2)$$

$$\frac{1}{Pr}\vec{V}^* \cdot (\nabla \vec{V}^*) = -\nabla P^* + \nabla^2 \vec{V}^* + RaT^* \vec{j} \qquad (3)$$

$$\vec{V}^* \cdot \nabla T^* = \nabla^2 T^* \qquad (4)$$

where $\vec{V}^*$ and T^* are the dimensionless velocity vector and temperature, respectively, and $Pr = \nu/\alpha$ and $Ra = g\beta\Delta TA^{3/2}/\nu\alpha$ are the Prandtl and Rayleigh numbers. The vector j is the unit vector in the direction of gravity (y-direction in Fig. 1). As shown in [3,4], the scales for the variables of Eqs. (2)–(4) are:

$$x^*, y^* = \frac{x, y}{\sqrt{A}}, \quad \vec{V}^* = \frac{\vec{V}}{\alpha/\sqrt{A}}, \quad T^* = \frac{T - T_c}{T_h - T_c} \quad \text{and} \quad P^* = \frac{P}{\rho\nu\alpha/A} \qquad (5)$$

Three types of boundary conditions were applied to the numerical domain: (i) non-slip to all internal surfaces, (ii) isothermal boundaries, T_h and T_c on the right and left walls of the outer enclosure, respectively, and (iii) thermal insulation in all other walls.

The constitutive equations were solved with Comsol Multiphysics® [38]. We used one of the solvers available in Comsol Multiphysics® namely UMFPACK [39]. The convergence criteria for all variables was set equal to 10^{-6}.

4. Model validation

The validation of the numerical implementation relied on the "benchmark" solutions proposed by de Vahl Davis [3] and de Vahl Davis and Jones [4]. These two articles provide detailed information