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Analysis of temperature gradient bifurcation in porous media – An exact solution

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ABSTRACT

The phenomenon of temperature gradient bifurcation in a porous medium is analyzed by studying the convective heat transfer process within a channel filled with a porous medium, with internal heat generation. A local thermal non-equilibrium (LTNE) model is used to represent the energy transport within the porous medium. Exact solutions are derived for both the fluid and solid temperature distributions for two primary approaches (Models A and B) for the constant wall heat flux boundary condition. The Nusselt number for the fluid at the channel wall is also obtained. The effects of the pertinent parameters such as fluid and solid internal heat generations, Biot number and fluid to solid thermal conductivity ratio are discussed. It is shown that the internal heat generation in the solid phase is significant for the heat transfer characteristics. The validity of the one equation model is investigated by comparing the Nusselt number obtained from the LTNE model with that from the local thermal equilibrium (LTE) model. The results demonstrate the importance of utilizing the LTNE model in the present study. The phenomenon of temperature gradient bifurcation for the fluid and solid phases at the wall for Model A is established and demonstrated. In addition, the temperature distributions for Models A and B are compared. A numerical study for the constant temperature boundary condition was also carried out. It was established that the phenomenon of temperature gradient bifurcation for the fluid and solid phases for the constant temperature boundary condition can occur over a given axial region.

1. Introduction

Convective heat transfer in porous media is encountered in a wide variety of industrial applications such as thermal energy storage, nuclear waste repository, electronic cooling, geothermal energy utilization, petroleum industry and heat transfer enhancement. A number of situations involve internal heat generation such as nuclear reactor applications, agricultural product storage, electronic cooling, or a solar air heater packed with a porous medium where the packed material provides the heat transfer enhancement and also acts as an absorbing media for the solar radiation [1].

Two primary models can be utilized for representing heat transfer in a porous medium: LTE and LTNE, which incorporates the temperature difference between the fluid and solid phases, thus resulting in different energy equations for the fluid and solid phases. Amiri and Vafai [2] employed a generalized model for the momentum equation and LTNE to investigate the forced convective heat transfer within a channel with a constant wall temperature. They investigated in detail the inertial and boundary effects, porosity variation, thermal dispersion and the validity of local thermal equilibrium as well as other pertinent effects. Amiri et al. [3] presented for the first time two primary approaches for the constant wall heat flux boundary conditions under the local thermal non-equilibrium condition in porous media. Based on the two-equation model (LTNE), and using one of the two primary approaches given in Amiri et al. [3], Lee and Vafai [4] investigated the forced convective flow through a channel filled with a porous medium subject to a constant heat flux, and derived exact solutions for both fluid and solid phase temperature fields. Marafie and Vafai [5] obtained analytical solutions for the fluid and solid phase temperature distributions for the forced convective flow through a channel filled with a porous medium with a constant heat flux boundary condition, in which the Brinkman–Forchhiemer-extended Darcy equation was used to obtain the velocity field. Alazmi and Vafai [6] presented a comprehensive analysis of the effect of using different boundary conditions for the case of constant wall heat flux under the local thermal non-equilibrium condition.

The main objective of the present study is to analyze the temperature gradient bifurcation phenomenon in porous media by investigating the heat transfer characteristics for convection through a channel filled with a porous medium, with internal heat generation in both the fluid and solid phases, and subject to a constant heat flux boundary condition. The analytical solutions for the fluid and solid phase temperature distributions and the Nusselt number at the channel wall are obtained. The effects of pertinent

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Nomenclature

Bi	$Bi = \frac{h_i \alpha H^2}{k_{s,eff}}$, Biot number defined by Eq. (13)
c_p	specific heat of the fluid [J kg^{-1} K^{-1}]
E	$E = \frac{Nu_1 - Nu}{Nu}$, error in the Nusselt number defined by Eq. (73)
h_i	interstitial heat transfer coefficient [W m^{-2} K^{-1}]
h_w	wall heat transfer coefficient defined by Eq. (25) [W m^{-2} K^{-1}]
h_{w1}	wall heat transfer coefficient calculated from one equation model, defined by Eq. (32) [W m^{-2} K^{-1}]
H	half height of the channel [m]
k	$k = \frac{k_{f,eff}}{k_{s,eff}}$, ratio of the fluid effective thermal conductivity to that of the solid, defined by Eq. (12)
$k_{f,eff}$	effective thermal conductivity of the fluid [W m^{-1} K^{-1}]
$k_{s,eff}$	effective thermal conductivity of the solid [W m^{-1} K^{-1}]
Nu	Nusselt number for the LTNE model, defined by Eq. (26)
Nu_1	Nusselt number for the LTE model, defined by Eq. (33)
q_w	heat flux at the wall [W m^{-2}]
Q	integrated internal heat transfer exchange between the solid and fluid phases [W m^{-2}]
Re	$Re = \frac{u(4H)}{\nu_f}$, Reynolds number
S_f	internal heat generation within the fluid phase [W m^{-3}]
S_s	internal heat generation within the solid phase [W m^{-3}]
T	temperature [K]
u	fluid velocity [m s^{-1}]
x	longitudinal coordinate [m]
y	transverse coordinate [m]

Greek symbols

$\Delta\theta$	non-dimensional temperature difference, $\Delta\theta = \theta_s - \theta_f$
α	interfacial area per unit volume of the porous medium [m^{-1}]
β	$\beta = \frac{S_s H}{q_w}$, parameter defined by Eq. (14)
β_1	$\beta_1 = \frac{\lambda}{(1+k)\tanh(\lambda)} - \frac{1}{1+k}$, parameter defined by Eq. (56)
β_2	$\beta_2 = -\frac{\lambda k}{(1+k)\tanh(\lambda)} - \frac{1}{1+k}$, parameter defined by Eq. (57)
β_3	$\beta_3 = -\frac{Bi}{3\left[1-\frac{1}{\lambda}\tanh(\lambda)\right]} - \frac{1}{1+k}$, parameter defined by Eq. (70)
η	non-dimensional transverse coordinate, defined by Eq. (6b)
ξ	non-dimensional axial length scale, defined by Eq. (51)
θ	non-dimensional temperature, defined by Eq. (6a) for Model A, defined by Eqs. (36) and (37) for Model B, or defined by Eq. (50) for constant temperature case
θ_b	non-dimensional bulk mean temperature for the LTE model
$\theta_{f,b}$	non-dimensional bulk mean temperature of the fluid, defined by Eq. (24)
λ	parameter defined by Eq. (21)
ρ	fluid density [kg m^{-3}]

Subscripts

f	fluid phase
s	solid phase
w	wall

Other symbols

$\langle\rangle$	average over the channel cross section

parameters such as internal heat generation, Biot number and thermal conductivity ratio are discussed. By comparing the Nusselt number obtained from the two-equation (LTNE) model with that from the one equation (LTE) model, the validity of the one equation model is investigated. In addition, the temperature distributions for two different approaches (Models A and B) for the constant wall heat flux boundary condition are compared. Furthermore, a numerical study for the constant temperature boundary condition was also carried out to investigate the temperature gradient bifurcation for that case. To the best of authors' knowledge, the present analysis of the temperature gradient bifurcation in porous medium is presented for the first time in the literature.

2. Modeling and formulation

The schematic diagram of the problem is shown in Fig. 1. Fluid flows through a rectangular channel filled with a porous medium subject to a constant heat flux boundary condition. We consider uniform but different internal heat generations in both the solid and fluid phases, S_s and S_f, respectively. The height of the channel is $2H$ and the heat flux applied at the wall is q_w. The following assumptions are invoked in analyzing this problem.

(1) The flow is steady and incompressible.
(2) Natural convection and radiative heat transfer are negligible.
(3) Properties such as porosity, specific heat, density and thermal conductivity are assumed to be constant.
(4) Thermally developed condition is considered and the fluid flow is represented by the Darcian flow model.

Based on these assumptions, the following governing equations are obtained from the works of Amiri et al. [2,3] employing the local thermal non-equilibrium model.

Fluid phase

$$k_{f,eff}\frac{\partial^2 T_f}{\partial y^2} + h_i\alpha(T_s - T_f) + S_f = \rho c_p u \frac{\partial T_f}{\partial x} \quad (1)$$

Solid phase

$$k_{s,eff}\frac{\partial^2 T_s}{\partial y^2} - h_i\alpha(T_s - T_f) + S_s = 0 \quad (2)$$

where T_f and T_s are the fluid and solid temperatures, u the fluid velocity, $k_{f,eff}$ and $k_{s,eff}$ the effective fluid and solid thermal conductivities, respectively, ρ and c_p the density and specific heat of the fluid, h_i the interstitial heat transfer coefficient, and α is the interfacial area per unit volume of the porous medium.

2.1. Boundary conditions [Model A]

When a solid substrate of finite thickness and high thermal conductivity is attached to the porous medium as shown in Fig. 1., the

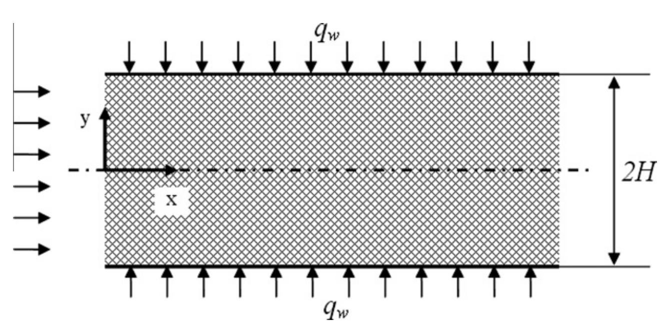

Fig. 1. Schematic diagram for flow through a channel filled with a porous medium and the corresponding coordinate system.