International Journal of Heat and Mass Transfer 54 (2011) 3347–3356

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Natural convection of liquid metal in a vertical annulus with lateral and volumetric heating in the presence of a horizontal magnetic field

S.C. Kakarantzas [a], I.E. Sarris [b,1], N.S. Vlachos [b,*]

[a] Physique Statistique et Plasmas, CP 231, Université Libre de Bruxelles, 1050 Brussels, Belgium
[b] Department of Mechanical Engineering, University of Thessaly, Athens Avenue, 38334 Volos, Greece

ARTICLE INFO

Article history:
Received 30 August 2010
Received in revised form 23 March 2011
Available online 16 April 2011

Keywords:
Free convection
MHD
Annular cavity
Lateral heating
Volumetric heating
Liquid metals

ABSTRACT

MHD free convection of a liquid metal is studied in a closed vertical annulus in which the upper and bottom walls are adiabatic while the cylindrical walls are kept at different temperatures. The flow is driven by two mechanisms; the temperature difference between the two cylindrical walls and the volumetric heating. A constant horizontal magnetic field is also imposed resisting the fluid motion. The laminar and turbulent regimes of the flow are assessed by performing three-dimensional direct numerical simulations. The results show that in the absence of the magnetic field, turbulent flow is developed in most of the cases, while as the magnetic field increases the flow becomes laminar. The highest temperature is found in the upper-central part of the annular cavity when the fluid is heated volumetrically, resulting in the creation of two convection currents as the hot fluid ascends in the central part and descends close to both colder walls. The Hartmann and Roberts layers developing near the walls normal and parallel to the magnetic field, respectively, are found to be responsible for the loss of axisymmetry of the present flow.

1. Introduction

Fusion reactors are most promising devices for the production of non-fossil fuel energy in very large quantities. In the last decades, numerous studies have dealt with the transfer of heat from the fusion reactor core which will be assisted by a liquid metal blanket. In such configurations, the increase of heat transfer and the decrease of the pressure drop along the flow channels is of great importance [1]. This is because very high pressure drops occur in duct and pipe flows under strong magnetic fields. As a consequence, the use of forced convection and more specifically liquid pumping for heat removal may become very expensive. In such situations, heat removal by efficient natural convection may be an economic alternative [2,3].

Various natural convection configurations have been proposed as more appropriate for enhancing heat transfer rates. More specifically, Li et al. [4] and Seriwaza et al. [5] suggested a geometry consisting of two coaxial cylinders where liquid metal is placed in the annular gap and a non-conductive gas (e.g. Helium) is flowing inside the inner cylinder. With this configuration the MHD effects are reduced significantly as the non-conductive gas can remove the heat without any additional pressure drop due to the magnetic field. Uda et al. [6] presented both experimental and numerical results of natural convection of a liquid metal, in a geometry consisting of a tube and a heater pin placed on the axis of the tube. A transverse magnetic field was applied and the mechanism of heat transfer was studied. The k–ε model was used for the numerical calculations, adapted for MHD natural convection flows. Moreover, Serizawa et al. [5] performed an experiment in a vertical tube using NaK/nitrogen flow and, applying an external transverse magnetic field, they demonstrated that heat transfer was non-axisymmetric. Similar results were found in the numerical study of Li et al. [4] for a liquid metal/gas flow in a vertical annulus under the effect of a transverse magnetic field.

In the present work a similar configuration to that used by Kakarantzas et al. [2,3] was considered where a liquid metal with Prandtl number of 0.0321 was placed between two coaxial vertical cylinders. The outer wall temperature was constant and greater than that of the inner. Internal volumetric heating and an external horizontal magnetic field were applied. The combined natural convection flow due to wall temperature difference and the internal heating is the additional feature studied here in relation to the previous work of Kakarantzas et al. [2].

* Corresponding author. Tel.: +30 24210 74094; fax: +30 24210 74085.
E-mail address: vlachos@mie.uth.gr (N.S. Vlachos).
[1] Present address: Technological & Educational Institute of Athens, Energy Technology Department, Ag. Spyridona 17, 12210 Egaleo, Athens, Greece.

0017-9310/$ - see front matter
doi:10.1016/j.ijheatmasstransfer.2011.03.051

Nomenclature

B_0	magnitude of the external magnetic field (kg/(s^{2}A))
g	gravity acceleration (m/s^2)
$Ha = RB_0\sqrt{\sigma/\rho\nu}$	Hartmann number
$\mathbf{J}$	electric current density (A/m^2)
k	fluid thermal conductivity (W/m.K)
L	height of the cylinders (m)
Nu_i	local Nusselt number at the inner cylinder
$\overline{Nu}_i$	azimuthally averaged Nusselt number along the inner cylinder
$\overline{Nu}_{i,tot}$	averaged Nusselt number on the inner cylinder
Nu_o	local Nusselt number at the outer cylinder
$\overline{Nu}_o$	azimuthally averaged Nusselt number along the outer cylinder
$\overline{Nu}_{o,tot}$	averaged Nusselt number on the outer cylinder
p	fluid pressure (Pa)
$Pr = \nu/\alpha$	Prandtl number
Q	volumetric heating rate (W/m^3)
$R = R_o - R_i$	annular gap (m)
$Ra_e = g\beta\Delta T\, R^3/\nu\alpha$	external Rayleigh number
$Ra_i = g\beta Q\, R^5/\nu\kappa\alpha$	internal Rayleigh number
R_i	radius of the inner cylinder (m)
R_o	radius of the outer cylinder (m)
r, z	radial and axial spatial coordinates
$S = Ra_i/Ra_e$	ratio of internal-to-external Rayleigh numbers
t	time (s)
T^*	temperature of the fluid
T_i	temperature of the inner cylinder (K)
T_o	temperature of the outer cylinder (K)
$\Delta T = T_o - T_i$	temperature difference between the cylinders (K)
$T = (T^* - T_i)/\Delta T$	non-dimensional temperature
u_r, u_θ, u_z	radial, tangential, and axial velocity components
$\mathbf{v}$	velocity vector

Greek letters

α	fluid thermal diffusivity (m^2/s)
β	fluid coefficient of thermal expansion (1/K)
θ	azimuthal angle
ν	fluid kinematic viscosity (m^2/s)
ρ	fluid density (kg/m^3)
σ	fluid electrical conductivity (s$^3A^2$/m^{2}kg)
Φ	electrical potential (m^{2}kg/s^{3}A)

2. Flow configuration and model description

A vertical annular container, as shown schematically in Fig. 1, was considered with an aspect ratio $L/R = 3$, where L is the height of the cylinders, $R = R_o - R_i$ is the annular gap, and R_o, R_i are the radii of the outer and inner cylinders, respectively, with $R_i = 0.2R$. The top and bottom walls were assumed adiabatic while the outer and inner walls were kept isothermal with the outer wall temperature T_o greater than that of the inner T_i. The annular container was filled with a low Prandtl number ($Pr = 0.0321$) electrical conducting fluid. Internal volumetric heating and an external horizontal magnetic field were applied. It must be noted that in Fig. 1, Q represents the volumetric heat generation, $\mathbf{g}$ is the gravity acceleration, and $\mathbf{B_o}$ the constant external magnetic field. All walls were assumed electrically insulated. The aspect ratio of $L/R = 3$ was selected since it corresponds to a fair balance between the actual design needs of the future fusion blanket and the efficiency of the direct numerical simulation method used in the present study.

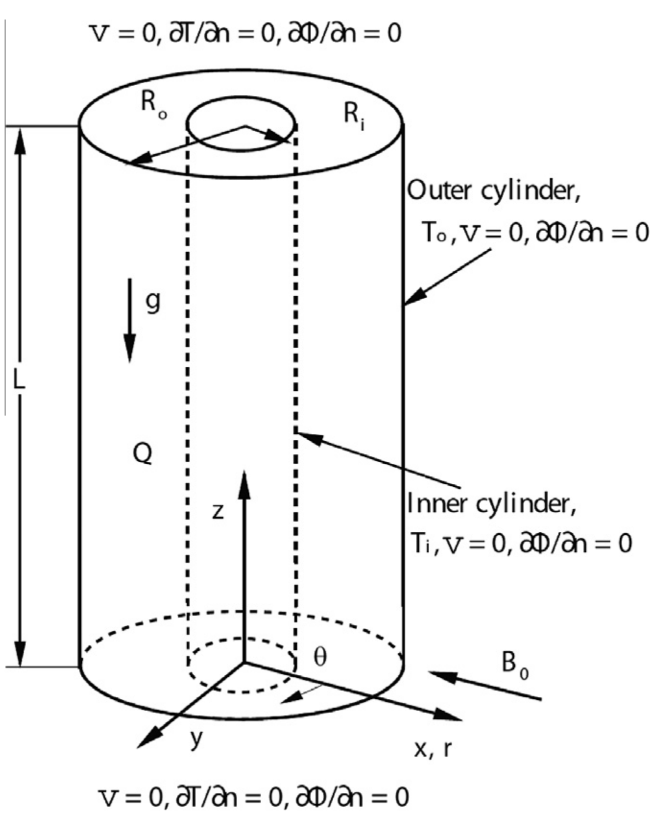

Fig. 1. Flow configuration and boundary conditions.

Table 1
Grid independence test ($Ra_e = 10^5$, $Ra_i = 0$, $Ha = 100$).

Grid	32 × 32 × 64	64 × 64 × 128	97 × 97 × 176
Nu	1.843	2.126	2.159
Error	14.6 %	1.5 %	–

The governing magnetohydrodynamic equations were non-dimensionlized using as characteristic quantities the annular gap R, the free fall velocity, $u_{ref} = \sqrt{g\beta R\Delta T}$, the pressure, $p_{ref} = \rho u_{ref}^2$, the time, $t_{ref} = R/u_{ref}$, and the electric potential, $\Phi_{ref} = B_0Ru_{ref}$, where, ρ is the density of the fluid and β its volumetric thermal expansion coefficient. The non-dimensional temperature T was calculated from the fluid temperature T^* using the relation $T = (T^* - T_i)/\Delta T$, where $\Delta T = T_o - T_i$ was the temperature difference between the outer and the inner cylindrical walls. Thus, the dimensionless equations become:

$$\nabla \cdot \mathbf{v} = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\nabla p + T\mathbf{k} + \left(\frac{Pr}{Ra_e}\right)^{\frac{1}{2}}\nabla^2\mathbf{v} + Ha^2\left(\frac{Pr}{Ra_e}\right)^{\frac{1}{2}}(\mathbf{J}\times\mathbf{B_0}), \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v}\cdot\nabla)T = \left(\frac{1}{PrRa_e}\right)^{\frac{1}{2}}\nabla^2 T + \frac{Ra_i}{(PrRa_e^3)^{\frac{1}{2}}}, \tag{3}$$

$$\nabla^2\Phi = \nabla\cdot(\mathbf{v}\times\mathbf{B_0}), \tag{4}$$

$$\mathbf{J} = -\nabla\Phi + \mathbf{v}\times\mathbf{B_0}, \tag{5}$$