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Estimation of front surface temperature and heat flux of a locally heated plate from distributed sensor data on the back surface

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ABSTRACT

We present a new method of solving the three-dimensional inverse heat conduction (3D IHC) problem with the special geometry of a thin sheet. The 3D heat equation is first simplified to a 1D equation through modal expansions. Through a Laplace transform, algebraic relationships are obtained that express the front surface temperature and heat flux in terms of those same thermal quantities on the back surface. We expand the transfer functions as infinite products of simple polynomials using the Hadamard Factorization Theorem. The straightforward inverse Laplace transforms of these simple polynomials lead to relationships for each mode in the time domain. The time domain operations are implemented through iterative procedures to calculate the front surface quantities from the data on the back surface. The iterative procedures require numerical differentiation of noisy sensor data, which is accomplished by the Savitzky–Golay method. To handle the case when part of the back surface is not accessible to sensors, we used the least squares fit to obtain the modal temperature from the sensor data. The results from the proposed method are compared with an analytical solution and with the numerical solution of a 3D heat conduction problem with a constant net heat flux distribution on the front surface.

1. Introduction

To conduct thermal measurements under harsh environment, it has been proposed that sensors be located away from direct contact with the environment and mathematical models be used to calculate the desired quantities from the sensor measurement data. Specifically, the front surface temperature can be determined indirectly by solving an inverse heat conduction problem (IHCP) [1–18] based on the transient temperature and/or heat flux measured on the back surface.

In our previous work [19], we applied the Laplace transform to the one-dimensional heat conduction problem. Algebraic equations relating temperature and heat flux on the front and back surfaces were obtained. Although these equations were no different from those given in [20–23], we proposed approximations to the transfer functions through which the front surface temperature and heat flux can be obtained from the data on the back surface through mathematical operations in the time domain. No complicated inverse Laplace transform is required. Following that work, we have replaced the ad hoc approximations to the transfer functions by a procedure which achieves better approximations through iterations [24]. The method was shown to be both accurate and easy to implement for one-dimensional IHCPs. However, it is formulated to handle one-dimensional IHCPs only.

The above method for one-dimensional IHCPs is too restrictive in measurement practices. We thus wish to generalize the method to make it applicable to realistic measurement problems. Since our motivation lies in measurement instrumentation, we have the flexibility of selecting the geometry and even boundary conditions that help to simplify the solution of the IHCP. We may restrict the geometry to that of a rectangular sheet. Furthermore, we assume that the sheet thickness is much smaller than the transverse dimensions. Therefore, heat conduction is mostly across the thickness. This special geometry and boundary condition allows us to introduce simplification from 3-dimension to 1-dimension. Integral transformation methods have been proposed in [22]. In this paper, we use Fourier series to convert the 3-dimensional heat conduction equation into a system of 1-dimensional equations. Each equation is solved by the method we proposed in [24].

In the following section, we present the reduction to the one-dimensional problem. Since the resulting one-dimensional problem contains an additional *source* term, we present in Section 3 the polynomial representation of the transfer function based on the Hadamard Factorization Theorem; the polynomial representation is a generalization of our prior result. From the polynomial representation of the transfer function, an iterative procedure is

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Nomenclature

c_{ps}	mass specific heat of the slab, J/(kg K)
E	total error
f	normalized heat flux, K
g	notation for the integrand
G	transfer function
k_s	thermal conductivity, W/(m K)
l_x	slab length, m
l_y	slab width, m
l_z	slab thickness, m
L_x	ratio of slab length to thickness, l_x/l_z
L_y	ratio of slab width to thickness, l_y/l_z
M	maximum mode number along the length direction
N	maximum mode number along the width direction
p	pole of a transfer function
q	heat flux, W/m^2
s	Laplace transform variable
t	time, s
t_c	characteristic time, s
T_s	temperature of the slab, K
u	unit step function
U	Laplace transform of temperature
x	spatial coordinate variable along the length, m
y	spatial coordinate variable along the width, m
z	spatial coordinate variable along the thickness, m
X	dimensionless spatial coordinate variable along the length direction
Y	dimensionless spatial coordinate variable along the width direction
Z	dimensionless spatial coordinate variable along the thickness direction

Greek symbols

α	thermal diffusivity, m^2/s
δ	Kronecker delta
ρ_s	density of the slab, kg/m^3
τ	dimensionless time
θ	temperature, K
Θ	Laplace transform of the temperature
Φ	Laplace transform of the normalized heat flux
ω	dimensionless frequency
ξ	dimensionless length variable

Subscripts

b	back surface quantity
f	front surface quantity
m	mode number along the length direction
n	mode number along the width direction

obtained. From the data on the back surface, we need to extract the data for each one-dimensional problem. In Section 4, we present the equations we need when the sensor distribution on the back surface is arbitrary. In Section 5, we validate our method for a single 3D mode by comparing our results with those from analytical solutions. In Section 6, we provide a comparison of our IHC results with a direct three-dimensional heat conduction calculation.

2. Mathematical model and reduction to one-dimensional problem

Although the formulation for a three-dimensional anisotropic heat conduction problem is given in [21], we give the following formulation for isotropic problem since it is the application we have in mind. Since our interest lies in the heat input and temperature on the front surface, we are at liberty in selecting the geometry and boundary conditions that are most convenient in instrumentation and subsequent data processing. Thus we consider the following heat conduction equation:

$$\rho_s c_{ps}\frac{\partial T_s}{\partial t} = k_s\left(\frac{\partial^2 T_s}{\partial x^2}+\frac{\partial^2 T_s}{\partial y^2}+\frac{\partial^2 T_s}{\partial z^2}\right) \quad (1)$$

where ρ_s, k_s and c_{ps} are the density, thermal conductivity and specific heat of the solid, all assumed to be independent of temperature. We assume adiabatic boundary conditions on the back surface and the four edge surfaces:

$$\frac{\partial T_s}{\partial x} = 0 \quad \text{for } x=0 \text{ and } x=l_x \quad (2)$$

$$\frac{\partial T_s}{\partial y} = 0 \quad \text{for } y=0 \text{ and } y=l_y \quad (3)$$

and

$$\frac{\partial T_s}{\partial z} = 0 \quad \text{for } z=l_z \quad (4)$$

On the front surface, the heat flux is specified:

$$-k_s\frac{\partial T_s}{\partial z} = q(x,y,t) \quad \text{for } z=0 \quad (5)$$

To simplify, introduce the following changes to dimensionless variables:

$$t=t_c\tau,\quad x=l_zX,\quad y=l_zY,\quad z=l_zZ \quad (6)$$

and to variables with dimensions of temperature (K)

$$q(x,y,t)=\frac{k_s}{l_z}f(X,Y,\tau) \quad (7)$$

$$T_s(x,y,z,t)=\theta(X,Y,Z,\tau) \quad (8)$$

where

$$t_c=\frac{\rho_s c_{ps} l_z^2}{k_s} \quad (9)$$

and $f(X, Y, \tau)$ is the normalized front surface heat flux (with unit K). With these substitutions, Eq. (1) becomes

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}+\frac{\partial^2\theta}{\partial Z^2} \quad (10)$$

with boundary conditions:

$$\frac{\partial\theta}{\partial X}=0 \quad \text{for } X=0 \text{ and } X=L_x \quad (11)$$

$$\frac{\partial\theta}{\partial Y}=0 \quad \text{for } Y=0 \text{ and } Y=L_y \quad (12)$$

$$\frac{\partial\theta}{\partial Z}=0 \quad \text{for } Z=1 \quad (13)$$

where $L_x = l_x/l_z$ and $L_y = l_y/l_z$. On the front surface, the heat flux is specified:

$$-\frac{\partial\theta}{\partial Z}=f(X,Y,\tau) \quad \text{for } Z=0 \quad (14)$$