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Effective surface modeling for momentum and heat transfer over rough surfaces: Application to a natural convection problem

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ABSTRACT

In this paper, we propose efficient and suitable effective surface models for steady laminar flows with heat transfer over rough surfaces. These models are developed in the frame of a domain decomposition method and consist in replacing the rough boundaries by effective smooth surfaces on which effective boundary conditions or wall laws are prescribed. The associated effective properties, namely the effective friction and heat transfer coefficients, are determined by the resolution of local *closure problems* over a representative pattern of the roughnesses. The impact of the flow parameters on these effective coefficients is analyzed, which allows to obtain useful estimates in some specific cases. Finally, two-dimensional numerical experiments are performed for a natural convection problem in a stamp shaped cavity to assess the validity of the proposed effective surface models. Throughout these tests, we also study numerically the impact of the position of the effective surface on momentum and heat transfers.

1. Introduction

Fluid flow with heat and mass transfer over rough surfaces appears in a large number of situations of practical interest including geophysical applications to study, for instance, ocean-ice exchanges, design of heat-exchangers or, in the aerospace context, the design of thermal barriers in reentry vehicle configurations. Transport phenomena over rough surfaces also take an important place in nuclear safety studies. As a first example, in the case of an accidental release of radioactive wastes, there is a strong interest to predict the dispersion of the radioactive elements in the near field release region (*e.g.* [29]). In this situation, the roughnesses that correspond to both the vegetal and the urban canopies remain much smaller than the near field region dimensions but play an important role in the dispersion and the deposition process and, therefore, in the source term evaluation. In the context of severe accident conditions, another situation of interest concerns the issue of molten core and concrete interaction where a molten mixture of core and vessel structures comes to interact with the concrete basement of the nuclear plant (*e.g.* [18,22]). During the concrete ablation process, roughnesses appear both at small scales due to concrete heterogeneities (mortar, silica aggregates, etc.) and at intermediate scales due to vigorous mixing (bubbling, compositional convection). Here again, the roughnesses remain smaller than the domain of interest which corresponds here to the reactor pit but are likely to alter the heat fluxes along the cavity shape and then the melting-throw time.

To tackle numerically these kinds of problems, two approaches are traditionally followed depending on the order of magnitude of the characteristic length scale ℓ of the roughness compared to the macro-scale dimension L of the domain of interest. On the one hand, when the two characteristic length scales have the same order of magnitude, it is still possible to describe the complicated roughness geometry since direct numerical simulations (DNSs) remain tractable. On the other hand, when $\ell \ll L$, DNSs become too expensive and cannot be achieved anymore for practical applications, especially for three-dimensional applications. In this case, the problem is characterized by two separated scales ℓ and L which are referred to as the micro- and the macro-scale, respectively. Then, the traditional way of solving these problems consists in describing the micro-scale average behavior by means of effective surface models, which replace the heterogeneous boundary by a smooth surface on which effective conditions or wall laws are prescribed. While such wall laws have been the subject of numerous studies in the case of turbulent flow over rough surfaces, the building of these laws in the case of laminar flow has received more recently a regain of attention from a theoretical point of view. A large number of studies have been devoted to the derivation of effective boundary conditions for laminar flows over rough surfaces using homogenization methodology (*e.g.* [2,6,25]). The obtained wall law, for the momentum equation, takes the form of a Navier boundary condition that relates the macro-scale tangential

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Nomenclature

Greek symbols

β	thermal expansion coefficient (K^{-1})
δ	effective surface position (m)
δ_0	fictitious surface position (m)
ϵ	$=\ell/L$ small parameter in asymptotic expansions (dimensionless)
Γ_{0i}	restriction of Γ_0 in Ω_i (dimensionless)
Γ_0	fictitious surface (dimensionless)
Γ_δ	effective surface (dimensionless)
Γ_{per}	periodic surface (dimensionless)
Γ_s	side surface (dimensionless)
Γ_{up}	upper surface (dimensionless)
Γ_w	rough surface (dimensionless)
κ	thermal diffusivity ($m^2\,s^{-1}$)
μ	dynamic viscosity (Pa s)
Ω	global domain (dimensionless)
Ω_1	sub-domain associated with the length scale L (dimensionless)
Ω_2	sub-domain associated with the length scale ℓ (dimensionless)
Ω_δ	effective domain (dimensionless)
Ω_i	pseudo-periodic unit cell (dimensionless)
ρ	density ($kg\,m^{-3}$)
τ_ℓ	$=\mathrm{Re}_\ell\mu^2/\rho\ell^2$ imposed shear stress (s^{-1})

Roman symbols

$\langle \mathrm{Nu} \rangle$	averaged Nusselt number (dimensionless)
$\langle \mathrm{Nu}_\delta \rangle$	averaged Nusselt number in the effective case (dimensionless)
$\langle \mathrm{Nu}_r \rangle$	averaged Nusselt number in the rough case (dimensionless)
$\langle \mathrm{Nu}_s \rangle$	averaged Nusselt number in the smooth case (dimensionless)
$\mathbf{A}$	closure variable for the velocity (dimensionless)
$\mathbf{a}$	closure variable for the pressure ($Pa\,s\,m^{-1}$)
$\mathbf{B}$	closure variable for the velocity (m)
$\mathbf{b}$	closure variable for the pressure (Pa s)
ΔT_ℓ	micro-scale temperature difference (K)
ΔT_L	macro-scale temperature difference (K)
ℓ	micro-scale length (m)
$\mathbf{n}$	exterior normal to Γ_w (dimensionless)
$\mathbf{n}_0$	exterior normal to Γ_0 (dimensionless)
Pe_ℓ	$=U\ell/\kappa$ micro-scale Péclet number (dimensionless)
Pe_L	$=UL/\kappa$ macro-scale Péclet number (dimensionless)
Pr	micro-scale Péclet number (dimensionless)
Ra_ℓ	$=\mathrm{Pr}\rho^2\beta\Delta T_\ell\, g\ell^3/\mu^2$ micro-scale Rayleigh number (dimensionless)
Ra_L	$=\mathrm{Pr}\rho^2\beta\Delta T_L\, gL^3/\mu^2$ macro-scale Rayleigh number (dimensionless)
Re_ℓ	$=\epsilon\rho U\ell/\mu$ micro-scale Reynolds number (dimensionless)
Re_L	$=\rho UL/\mu$ macro-scale Reynolds number (dimensionless)
$\tilde{\mathbf{u}}_2$	spatial deviation velocity ($m\,s^{-1}$)
$\tilde{p}_2$	spatial deviation pressure (Pa)
$\tilde{T}_2$	spatial deviation temperature (K)
$\mathbf{u}$	velocity defined in Ω ($m\,s^{-1}$)
$\mathbf{u}_{1,0}$	spatial restriction of velocity on Γ_0 ($m\,s^{-1}$)
$\mathbf{u}_{1,2}$	macro-scale velocity defined in Ω_2 ($m\,s^{-1}$)
c	closure variable for the temperature (dimensionless)
$C_{f,\delta}$	effective friction coefficient defined on Γ_δ ($Pa\,s\,m^{-1}$)
C_f	effective friction coefficient defined on Γ_0 ($Pa\,s\,m^{-1}$)
d	closure variable for the temperature (m)
g	acceleration due to gravity ($m\,s^{-2}$)
h	effective heat transfer coefficient defined on Γ_0 ($m\,s^{-1}$)
h_δ	effective heat transfer coefficient defined on Γ_δ ($m\,s^{-1}$)
L	macro-scale length (m)
p	pressure defined in Ω (Pa)
$p_{1,0}$	spatial restriction of pressure on Γ_0 (Pa)
$p_{1,2}$	macro-scale pressure defined in Ω_2 (Pa)
T	temperature defined in Ω (K)
$T_{1,0}$	spatial restriction of temperature on Γ_0 (K)
$T_{1,2}$	macro-scale temperature defined in Ω_2 (K)
T_r	reference temperature defined in Ω (K)
T_w	temperature defined on Γ_w (K)
U	magnitude of the macro-scale velocity ($m\,s^{-1}$)
x	abscissa (m)
y	ordinate (m)

velocity to the macro-scale shear rate through an effective friction coefficient, which has to be determined by solving a local boundary value problem. In microfluidics, the derivation of effective conditions has also been the subject of numerous studies for laminar flows over rough and hydrophobic surfaces (*e.g.* [30]). In such applications involving nano-scales geometries, the corresponding effective conditions are referred to as the effective stick–slip boundary condition. This latter example, however, may be considered as slightly different of the exposed problem since an additional length-scale may play an important role, namely, the particle mean free path. For flows over porous media, which present some similarities with the rough surface case, the development of effective boundary conditions has also been revisited since the semi-empirical relation of Beavers and Joseph [7] through different up-scaling methods such as the volume averaging approach [17], matched asymptotic expansions [10] and multi-scale asymptotics [4,19] to cite among others.

In the present paper, we are interested in building effective boundary conditions for steady laminar flows with heat transfer by a domain decomposition method. As this study constitutes a first step towards more complicated problems such as those listed above for nuclear safety study analysis, this relatively simplified problem constitutes a model problem coupling flow with the transport of a non-passive scalar. Such a problem has received considerable attention (*e.g.* [3,20,36]) to investigate the impact of roughnesses on heat transfer. It has been emphasized that the presence of roughnesses leads to an enhancement of the total heat transfer rate, which is influenced by different parameters such as, for example, the wavelength and the amplitude of the roughnesses.

To be specific about the problem under consideration, we focus in this work on steady laminar natural convection flows over rough surfaces in the frame of the Boussinesq approximation and we refer to the Fig. 1 in which we have shown the geometrical features of the domain Ω with rough boundary Γ_w. For the sake of simplicity, the developments that follow are made in the two-dimensional case but they can be obviously generalized in the three-dimensional case. The boundary value problem used to describe momentum and heat transfer is given by

$$\rho(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mu\nabla^2\mathbf{u} - \rho\beta(T - T_r)\mathbf{g} \quad \text{in } \Omega \tag{1a}$$

$$\nabla\cdot\mathbf{u} = 0 \quad \text{in } \Omega \tag{1b}$$

$$\mathbf{u}\cdot\nabla T = \kappa\nabla^2 T \quad \text{in } \Omega \tag{1c}$$

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_w \tag{1d}$$

$$(\text{BC1}) \quad T = T_w \quad \text{on } \Gamma_w \tag{1e}$$

$$(\text{BC2}) \quad \mathbf{n}\cdot\nabla T = 0 \quad \text{on } \Gamma_w \tag{1f}$$

where p is the deviation from the hydrostatic pressure, μ is the dynamic viscosity, β the thermal expansion coefficient, κ the thermal diffusivity and T_r denotes some reference temperature.