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Technical Note

Energy losses and heat transfer enhancement in transversally corrugated pipes

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ABSTRACT

Wall friction, temperature distribution and heat transfer through pipe walls are investigated in forced convection with Newtonian fluids in pressure gradient driven hydrodynamically and thermally fully developed steady laminar flow in transversally corrugated pipes. Novel analytical solutions derived via the epitrochoid conformal mapping are presented for the velocity and temperature fields. Analytical results are compared with numerical solutions obtained using the finite volume technique. The effect of the corrugation amplitude and the number of waves on the friction factor, the temperature distribution and the Nusselt number is discussed.

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1. Introduction

Hydrodynamically and thermally fully developed laminar flow in ducts is very common in a number of industrial applications and has been extensively investigated. The same cannot be said however when it comes to non-circular ducts. But the importance of analytical and experimental results for laminar forced convection in non-circular ducts to the design of compact heat exchangers, for example among others, cannot be overstated. Unconventional channel geometries are encountered in a variety of compact heat exchangers and other industrial devices. For instance, corrugated and double-trapezoidal ducts are central to the design of honeycomb wheels [1], and lamella type compact heat exchangers [2,3], respectively. The latter are used extensively in pulp and paper, alcohol, petrochemical and other chemical industries.

Early works on non-circular ducts have been reviewed by Shah and London [4], and Shah and Bhatti [5]. Heat transfer with power-law fluids in arbitrary cross-sectional ducts and in non-circular geometries found in recent designs of compact heat exchangers has been investigated by Lawal and Mujumdar [6] and Manglik and Bergles [7], respectively. Other flow geometries investigated in the literature include the double-sine [8], the circular-segment [9], the semi-circular [10,11], the eccentric-annular [12], the rhombic [13], the regular polygonal [14], and several other unusual duct shapes considered by Sunden and Faghri [15]. Siginer and co-workers consider the flow of inelastic non-Newtonian and viscoelastic fluids in arbitrary geometries, Letelier and Siginer [16], Letelier et al. [17] and Siginer and Letelier [18]. Analytical expressions for the velocity and shear-stress fields for the fully developed pressure gradient driven flow in tubes of arbitrary cross-section for the class of viscoinelastic–viscoplastic fluids are derived in [16]. Analytical solutions for the longitudinal and transversal fields in pipes of arbitrary cross-section for a multiple integral fluid of the memory type are developed in [17,18]. Corrugated geometries may present a highly desirable alternative in heat transfer enhancement and to our knowledge have not received the attention they should have commanded. Turian and Kessler [19] assume a tube boundary described by $r = R[1 + \varepsilon f(\theta)]$ and use ε as the perturbation parameter to obtain an approximate solution for the velocity field, but do not address the heat transfer problem. Similar work for microtubes has been carried out by Duan and Muzychka [20].

In this paper the forced convection of Newtonian fluids in straight transversally corrugated pipes under steady, incompressible, fully developed laminar flow is investigated to determine the effect of the corrugation and the number of boundary waves on the friction factor, temperature profile and the wall heat transfer coefficient. An exact analytical solution for the velocity field via the epitrochoid conformal mapping is presented. A new analytical solution is derived for the temperature distribution and consequently for the heat transfer coefficient. The analytical work is

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Nomenclature

A	area
n	waviness
w	axial velocity
$\bar{w}$	average velocity
a	characteristic length scale of the duct cross section
f	Fanning friction factor $= \frac{2\bar{\tau}_w}{a\bar{w}^2}$
$\tilde{P}$	perimeter
P	pressure gradient
p	pressure
D_h	hydraulic diameter = $4A/\tilde{P}$
$\dot{q}_w$	wall heat flux
$\bar{\dot{q}}_w$	average wall heat flux
$\bar{h}$	average heat transfer coefficient
T	temperature
$\bar{T}_w$	average wall temperature
T_m	bulk temperature
Nu	Nusselt number $= \frac{D_h\bar{h}}{k}$
Q	flow rate
k	thermal conductivity
c_p	specific heat
x, y, z	coordinates in the flow domain ζ
Re	Reynolds number
r	corrugated boundary radius

Greek symbols

ρ_0	density
ε	corrugation amplitude
ε_c	critical corrugation amplitude
μ	viscosity
ζ	complex function in the flow domain
ξ	complex function in the mapped computational domain
ρ, φ, z	coordinates in the mapped computational domain ξ
$\bar{\tau}_w$	average wall shear stress

Subscripts

b	bulk
w	wall

complemented by and validated through numerical simulations using the finite volume technique. The agreement is excellent between the analytical and numerical solutions.

2. Analytical solution

The flow is considered to be laminar and steady fully developed both thermally and hydrodynamically. Fluid properties are assumed constant thus constitutive parameters do not depend on temperature. The case of the constant wall heat flux is investigated. It is also assumed that Fourier's law of heat conduction is valid and that internal energy and thermal conductivity do not depend explicitly on the velocity gradient or other kinematic quantities. With these assumptions the hydrodynamic and thermal problems become fully decoupled. The solution for the velocity field is presented followed by the solution for the thermal problem which is the main focus of the present report.

2.1. Analytical solution for the velocity field

Poiseuille flow of a Newtonian fluid in a tube of average radius a is considered. Due to the symmetries of the problem the velocity field is unidirectional $v = [0, 0, w(x, y)]$, and the Navier–Stokes equations can be simplified,

$$\frac{\partial p}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{2.1}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{8w_0}{a^2}, \quad w_0 = \frac{a^2 P}{8\mu}, \quad P = -\frac{dp}{dz} \tag{2.2}$$

The flow domain is mapped onto the domain inside the unit circle via the epitrochoid conformal mapping, Muskhelishvili [21],

$$\zeta = x + iy = \xi + \frac{\varepsilon\xi^{n+1}}{a^n} \tag{2.3}$$

with $n \geqslant 1$ and ε representing constants. The transformed domain ξ is defined as

$$\xi = \rho e^{i\varphi}, \quad 0 \leqslant \rho \leqslant a, \quad -\pi \leqslant \varphi \leqslant \pi \tag{2.4}$$

or in terms of Cartesian coordinates x and y,

$$\begin{aligned} x &= \rho\cos\varphi + \varepsilon\frac{\rho^{n+1}}{a^n}\cos[(n+1)\varphi] \\ y &= \rho\sin\varphi + \varepsilon\frac{\rho^{n+1}}{a^n}\sin[(n+1)\varphi] \end{aligned} \tag{2.5}$$

This map is conformal provided $|\varepsilon|(n + 1) < 1$; it maps the inside of the unit circle in the ξ-plane into a compact region in the ζ-plane with corrugated boundary of n corrugations and of radius r given through

$$r^2 = \rho^2 + \varepsilon^2 a^{-2n}\rho^{2n+2} + 2\varepsilon a^{-n}\rho^{n+2}\cos(n\varphi) \tag{2.6}$$

Since the radius varies between $1 \pm \varepsilon$ and there are n peaks ε can be regarded as the amplitude and $2\pi/n$ the wavelength of the corrugation. Two examples of corrugated geometries for ($n = 3, \varepsilon = 0.23$) and ($n = 5, \varepsilon = 0.13$) are given in Fig. 1.

Under the epitrochoid transformation, Eq. $(2.2)_1$ and the related boundary conditions become,

$$\begin{aligned} &\frac{1}{J}\left(\frac{\partial w}{\partial \rho} + \rho\frac{\partial^2 w}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial^2 w}{\partial \varphi^2}\right) = 8\frac{w_0}{a^2} \\ &J = \rho\{1 + \varepsilon^2(n+1)^2 a^{-2n}\rho^{2n} + 2\varepsilon(n+1)a^{-n}\rho^n\cos(n\varphi)\} \end{aligned} \tag{2.7}$$

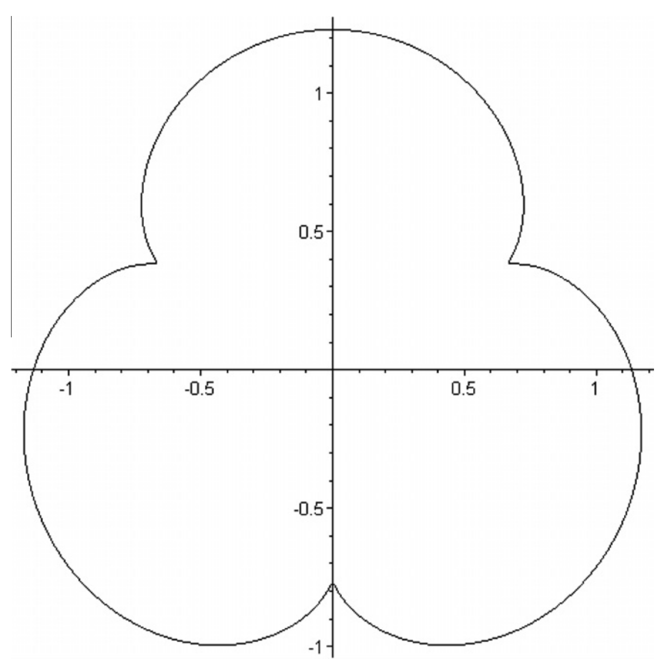

Fig. 1a. Flow domain for $n = 3$ and $\varepsilon = 0.23$.

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