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Mixed convection with heating effects in a vertical porous annulus with a radially varying magnetic field

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ABSTRACT

Fully developed parallel flow in an annular region filled with a porous medium surrounding an electric cable is investigated. The effects of buoyancy and MHD force as well as the heat generation due to Joule heating and viscous dissipation are taken into account. The mixed convection seepage flow is analyzed according to Darcy law and to Boussinesq approximation. Buoyancy effect is modelled by setting the isoflux wall temperature as the reference temperature. As a consequence of this choice, the local momentum and energy balance equations and the boundary conditions can be written in a dimensionless form that defines an initial value problem instead of a boundary value problem. The initial value problem is solved both by an analytical series method and by numerical integration. The effect of the radially varying magnetic field on the fluid velocity and temperature distributions is analyzed. It is shown that a significantly strong magnetic force tends to inhibit the flow even for a high hydrodynamic pressure gradient.

1. Introduction

The effects of an external magnetic field on convection flows in porous media has gained through the years an increasing attention, as pointed out in the comprehensive review by Nield and Bejan [1]. The interest in this field is due to the wide range of applications either in engineering and in geophysics, such as the optimization of the solidification processes of metals and metal alloys, the study of geothermal sources, the treatment of nuclear fuel debris, the control of underground spreading of chemical wastes and pollutants and the design of MHD power generators.

The analysis of hydromagnetic flows in porous media has been the subject of several recent papers [2–14]. These investigations can be considered as theoretical extensions of the deep knowledge reached in the last decades regarding MHD effects in fluid dynamics and convection heat transfer.

Most of the published papers on convection and porous media under the action of a magnetic field deal with external flows and consider cases such that the magnetic field is uniform. Kumari et al. [3] employ the numerical Keller box method to study the mixed convection in a porous medium around a vertical wedge. The boundary layer equations are solved by these authors considering the Brinkman model with inertia term for momentum transport and by taking into account both the effects of Joule heating and viscous dissipation in the energy balance. Chamkha and Quadri [4] consider hydromagnetic natural convection from a horizontal permeable cylinder and obtain a numerical solution of the nonsimilar boundary layer problem by using a finite difference method. El-Amin [7] investigates external free convection from either a horizontal plate or a vertical plate with uniform heat flux. The local balance equations are written with reference to power-law fluid flow in a porous medium, transformed introducing a similarity variable and solved through a fourth-order Runge–Kutta method with shooting technique. Postelnicu [8] analyzes simultaneous heat and mass transfer by natural convection from a vertical flat plate with uniform temperature in an electrically conducting fluid saturated porous medium. This author uses the Darcy–Boussinesq model including Soret and Dufour effects and solves numerically the similar boundary layer equations.

An interesting research work on analytical solutions for MHD effects in heat and momentum transfer involving either Newtonian or non-Newtonian fluids has been recently performed by Hayat and coworkers [10–12]. For instance, in Hayat et al. [10], the authors obtain exact solution for the MHD pipe flow of a Burgers' fluid in a porous medium by means of Fourier transform method. These authors adopt a modified Darcy's relationship and treat as special cases Oldroyd-B, Maxwell, second grade and Navier–Stokes fluid models. In a very recent paper, Khan et al. [11] treat incompressible Oldroyd-B fluid transient flow in a porous duct of rectangular cross-section, in the presence of an applied uniform magnetic field normal to the flow direction.

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Nomenclature

$A_n, \widetilde{A}_n, B_n, G_n$	series coefficients
$\mathbf{B}$	magnetic field
B_0	constant magnetic field, Eq. (1)
Br	Brinkman number, Eq. (11)
$\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_Z$	angular and axial unit vectors
$\mathbf{g}$	gravitational acceleration
g	modulus of the gravitational acceleration
$G(r)$	dimensionless function, Eq. (32)
Gr	Grashof number, Eq. (11)
I	electric current
k	thermal conductivity of the fluid
K	permeability
K_{eff}	effective permeability, Eq. (7)
Li_2	Euler's dilogarithm function
M	Hartmann number, Eq. (8)
Nu	Nusselt number, Eq. (18)
p	pressure
P	hydrodynamic pressure, $p + \rho g Z$
q_{w}	heat flux at the internal wall
r	dimensionless radial coordinate, Eq. (11)
R	radial coordinate
R_1	internal radius
R_2	external radius
Re	Reynolds number, Eq. (11)
t	dimensionless temperature, Eq. (11)
T	temperature
T_{ref}	reference temperature, Eq. (4)
T_{w}	temperature of the external boundary
$u, \tilde{u}$	dimensionless velocity, Eqs. (11) and (43)
$u_{\text{m}}, \tilde{u}_{\text{m}}$	dimensionless average velocity, Eq. (20) and (43)
$\mathbf{U}$	velocity
U	axial velocity component
U_{m}	average velocity in a transverse cross-section, Eq. (19)
U_{ref}	reference velocity, Eq. (13)
Z	axial coordinate

Greek symbols

β	volumetric coefficient of thermal expansion
γ	radial aspect ratio, Eq. (11)
ΔT	reference temperature difference, Eq. (12)
η	dynamic viscosity
θ	angular coordinate
Λ	dimensionless parameter, Eq. (43)
μ_0	magnetic permeability of vacuum
Ξ	dimensionless parameter, Eq. (11)
ρ	mass density
σ	electric conductivity of the fluid
ϕ	porosity

Bhadauria [13] analyzes the thermal instability of Brinkman model flow in an electrically conducting fluid saturated porous medium confined between two horizontal walls. The presence of an applied vertical magnetic field and uniform rotation around a vertical axis are considered.

In the present paper, we will study a physically conceivable system sufficiently simple to be described by a completely analytical solution of the governing equations. In the literature, the main value of analytical solutions is that they can be used as benchmarks to test numerical codes designed to study actual industrial devices, usually too complicated to be described by an analytical solution. There is also a second important value of analytical solutions: their simple and basic character allows one to investigate the fundamental aspects of a given physical phenomenon.

The aim of the present paper is to perform a study of combined forced and free flow of an electrically conducting fluid in a vertical annular porous medium surrounding a straight cylindrical electric cable. For instance, a technical system that can be approximately described according to this model is an electric cable surrounded by moist soil, especially in the case where salt water is present. The cable generates a transverse radially varying magnetic field in the fluid and yields to the fluid saturated porous medium a uniform heat input through the internal boundary of the annulus. Thus, the analysis will refer to an internal flow and to a non-uniform magnetic field. The Darcy–Boussinesq model is considered and the non-linear equations will be solved both analytically with a power series method and numerically by predictor–corrector Adams method.

2. Mathematical model

Let us consider a fluid saturated vertical porous annulus with internal radius R_1 and external radius R_2, that surrounds a very long straight cable with radius R_1 carrying a constant electric current I (see Fig. 1). Let us assume that the cable is electrically insulated and that the magnetic field created by the current I is not appreciably modified by the feedback field induced by the fluid flow in the porous medium. The fluid flow in the porous medium is steady, laminar, parallel and fully developed, so that the seepage velocity can be expressed as $\mathbf{U} = U\hat{\mathbf{e}}_Z$, where $\hat{\mathbf{e}}_Z$ is the unit vector in the axial direction Z. Biot–Savart law implies that the magnetic field $\mathbf{B}$ generated by the electric current I is given by

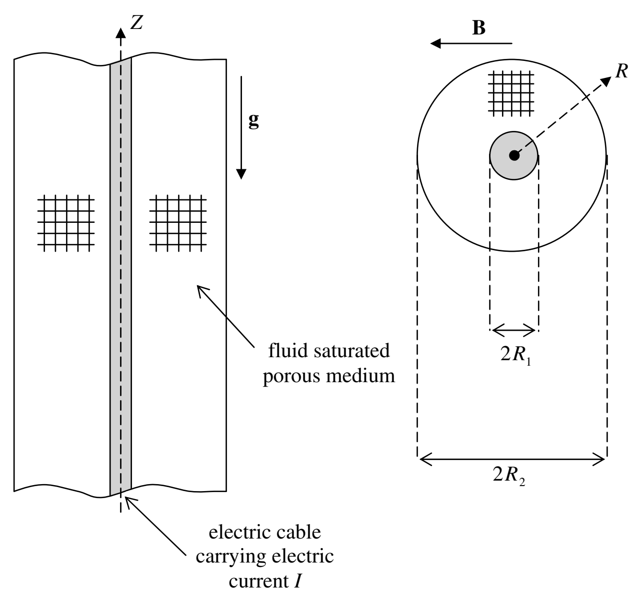

Fig. 1. Drawing of the vertical annulus.

$$\mathbf{B} = B_0 \frac{R_1}{R} \hat{\mathbf{e}}_\theta, \quad B_0 = \frac{\mu_0 I}{2\pi R_1}, \tag{1}$$

where $\hat{\mathbf{e}}_\theta$ is the unit vector in the azimuthal direction θ. The flow in the porous medium is described according to Darcy law and the effect of buoyancy is considered through the Boussinesq approximation [1,15]. Therefore, the mass balance equation requiring $\mathbf{U}$ to be