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# Mixed convection MHD flow in a vertical channel: Effects of Joule heating and viscous dissipation

A. Barletta *, M. Celli

*Università di Bologna, Dipartimento, di Ingegneria Energetica, Nucleare e del Controllo Ambientale (DIENCA), Laboratorio di Montecuccolino, Via dei Colli, 16, Bologna I-40136, Italy*

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ABSTRACT

Combined forced and free flow in a vertical channel with an adiabatic wall and an isothermal wall is investigated. The laminar, parallel and fully developed regime is considered. A uniform horizontal magnetic field is assumed to be applied to the fluid. The local balance equations are written in a dimensionless form and solved by taking into account the effects of Joule heating and viscous dissipation. The solutions are obtained both analytically by a power series method and numerically. The dimensionless governing parameters affecting the velocity and temperature profiles are the Hartmann number and the ratio between the Grashof number and the Reynolds number. Dual solutions are shown to exist for every value of the Hartmann number within a bounded range of the ratio between the Grashof number and the Reynolds number. Outside this range, no parallel flow solutions of the problem exist.



## 1. Introduction

Some authors [1–5] have pointed out that the laminar solution of steady natural or mixed convection problems may be not unique. In particular, the existence of dual solutions of boundary value problems describing external flows has been shown in Refs. [1–4]. Recently [5], the following result has been obtained: for fully developed laminar flow in a plane vertical channel with isothermal walls at the same temperature, the local balance equations admit two different solutions for any given value of the volume flow rate. Moreover, in the case of upward mean flow, there exist a maximum value of the volume flow rate above which no laminar parallel flow solution is admitted [5]. The presence of dual solutions is due to the nonlinearity of the balance equations produced by viscous dissipation. Situations in which the fully developed laminar flow in a vertical channel is described by nonlinear balance equations occur, for instance, when a magnetohydrodynamic force is present and the thermal generation due to Joule effect is non negligible. In recent years, much attention has been devoted to the study of magnetohydrodynamic effects on natural and mixed convection flows [6–11]. Indeed, convective flows in the presence of magnetic fields occur in many technical applications, such as, for instance, the optimization of industrial casting of metals [12]. In particular, in [6] an analytical solution for the natural convection in a two-dimensional rectangular cavity has been determined, in the presence of a vertical magnetic field. Pan and Li [7] have studied the mixed convection in a vertical plane channel with a horizontal magnetic field, in conditions of microgravity with a gravitational acceleration that oscillates in time with a sinusoidal law (g-jitter effect). The mixed convection flow in a horizontal circular duct in the presence of a uniform vertical magnetic field has been studied numerically in Ref. [8]. An experimental study on the natural convection of a $Na^{22}K^{78}$ alloy in a cavity with a rectangular section, in the presence of a vertical magnetic field, has been presented by Burr and Müller [9]. These authors have shown that the magnetic field produces a systematic decrease of heat fluxes in the fluid. In Ref. [10], the authors study the mixed convection in a vertical channel by considering the effects of viscous dissipation and of Joule heating. They determine the velocity and the temperature distribution both analytically, by means of a perturbation expansion, and numerically, by a finite difference method. Sposito and Ciofalo [11] have obtained analytical solutions of the local balance equations for fully developed mixed convection in a vertical plane channel, by considering isothermal walls and several electric boundary conditions.

In this paper, the steady laminar flow of an electrically conducting fluid in a plane vertical channel is considered. The velocity field is parallel to the gravitational acceleration and is orthogonal to the external magnetic field; the latter is uniform and is not influenced by the fluid flow. One of the channel walls is adiabatic, while the other is isothermal. The local balance equations are nonlinear and the boundary value problem, solved analytically, presents two different solutions for each value of the prescribed pressure gradient, provided that the latter lies within a bounded range outside which no laminar and parallel solution exists.

* Corresponding author.
*E-mail address:* antonio.barletta@mail.ing.unibo.it (A. Barletta).

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**Nomenclature**

| | |
|---|---|
| $A_n$ | series coefficients |
| $\vec{B}$ | magnetic induction field |
| $B$ | modulus of $\vec{B}$ |
| $\vec{E}$ | induced electric field |
| $\vec{f}$ | magnetic body force |
| $\vec{g}$ | acceleration due to the gravity |
| $g$ | modulus of $\vec{g}$ |
| $Gr$ | Grashof number, Eq. (12) |
| $\vec{J}$ | current density |
| $k$ | thermal conductivity of the fluid |
| $L$ | channel width |
| $M$ | Hartmann number, Eq. (12) |
| $P$ | hydrodynamic pressure, $p + \rho g X$ |
| $q_g$ | power generated per unit volume |
| $Re$ | Reynolds number, Eq. (12) |
| $T$ | temperature |
| $T_w$ | wall temperature |
| $T_{ref}$ | reference temperature |
| $u$ | dimensionless velocity |
| $\vec{U}$ | velocity |
| $U$ | vertical velocity component |
| $U_r$ | reference velocity, Eq. (13) |
| $X$ | vertical Cartesian coordinate |
| $Y$ | horizontal Cartesian coordinate |
| $y$ | dimensionless coordinate, Eq. (12) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | slope of $u(y)$ at $y = 0$, Eq. (16) |
| $\beta$ | volumetric coefficient of thermal expansion |
| $\Delta T$ | temperature scale, Eq. (13) |
| $\phi$ | dimensionless flux, Eq. (25) |
| $\Lambda$ | dimensionless parameter, Eq. (12) |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\vartheta$ | dimensionless temperature, Eq. (12) |
| $\rho$ | mass density |
| $\sigma$ | electric conductivity |
| $\tau_{XY}$ | shear stress applied at the wall |

*Superscripts, subscripts*

| | |
|---|---|
| $'$ | derivative with respect to $y$ |
| $(\pm)$ | positive/negative threshold value |
| cr | threshold value |
| f | first branch of solutions |
| JH | Joule heating |
| l | left branch of solutions |
| r | right branch of solutions |
| s | second branch of solutions |
| VD | viscous dissipation |

## 2. Governing equations

We consider the steady laminar flow of an electrically conducting fluid of electric conductivity $\sigma$ in a vertical parallel plane channel of width $L$. The $X$-axis of the coordinate system is opposite to the gravitational acceleration $\vec{g}$ and the $Y$-axis is perpendicular to the channel walls which are assumed to be impermeable (see Fig. 1). Flow is parallel so that the velocity is directed along the $X$-axis. The left wall (at $Y = 0$) is insulated (adiabatic) and the right one (at $Y = L$) is kept at the constant temperature $T_w$. The fluid motion is driven simultaneously by an applied pressure gradient, the buoyancy force and the MHD force due to a uniform external magnetic induction field $\vec{B}$ perpendicular to the channel walls. No external electric field is applied. Moreover, the magnetic Reynolds number is so small that the magnetic field induced by the moving fluid is negligible with respect to the external magnetic field.

The induced electric field is $\vec{E} = \vec{U} \times \vec{B}$, so that the current density is given by

$$\vec{J} = \sigma\vec{E} = \sigma\,\vec{U} \times \vec{B}, \tag{1}$$

where $\sigma$ is the electric conductivity of the fluid, which will be considered as constant. Since $\vec{B}$ is orthogonal to $\vec{U}$ the magnetic body force per unit volume can be expressed as

$$\vec{f} = -\sigma B^2 \vec{U}. \tag{2}$$

The power per unit volume generated by Joule effect is

$$q_g = \vec{J} \cdot \vec{E} = \sigma(\vec{U} \times \vec{B}) \cdot (\vec{U} \times \vec{B}) = \sigma B^2 U^2. \tag{3}$$

Let us denote by $\rho$ the density at the reference temperature $T_{ref}$. The fully developed parallel flow condition and the uniform wall temperature imply that the fluid velocity $U$ along $X$ and the fluid temperature $T$ depend only on $Y$, the hydrodynamic pressure $P = p + \rho g X$ depends only on $X$ and $dP/dX$ is constant. We also assume that the Boussinesq approximation holds and that both the Joule heating and the heat generation by viscous dissipation must be taken into account. The momentum and energy equations can be expressed as

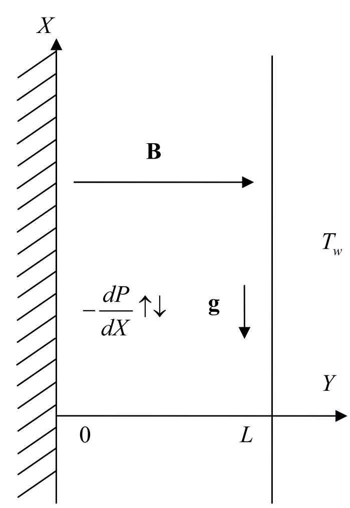


**Fig. 1.** Drawing of the vertical channel.

$$\mu \frac{d^2U}{dY^2} - \sigma B^2 U + \rho g \beta (T - T_{ref}) - \frac{dP}{dX} = 0, \tag{4}$$

$$k\,\frac{d^2T}{dY^2} + \sigma B^2 U^2 + \mu \left(\frac{dU}{dY}\right)^2 = 0. \tag{5}$$

The reference temperature $T_{ref}$ is chosen equal to the temperature $T(0)$ of the adiabatic wall. According to the Boussinesq approximation, the values of $\rho$, $\mu$, $\beta$, $k$ and $\sigma$ are taken at the reference temperature $T_{ref}$. The no slip conditions and the prescribed thermal boundary conditions are given by

$$U(0) = U(L) = 0, \tag{6}$$

$$\left.\frac{dT}{dY}\right|_{Y=0} = 0, \quad T(L) = T_w. \tag{7}$$