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Developing slip-flow and heat transfer in trapezoidal microchannels

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ABSTRACT

Simultaneously developing velocity and temperature fields in the slip-flow regime are investigated numerically in trapezoidal microchannels with constant wall temperatures. A wide range of channel aspect ratios ($0.25 \leqslant \alpha \leqslant 2$) and side angles ($30^\circ \leqslant \phi \leqslant 90^\circ$) are considered in the Reynolds number range $0.1 \leqslant Re \leqslant 10$. A control-volume based numerical method is used to solve the Navier–Stokes and energy equations with velocity-slip and temperature-jump at the walls. As characterized by the Knudsen number ($Kn \leqslant 0.1$), the effects of rarefaction on the key flow features are examined in detail. Major reductions in the friction and heat transfer coefficients are observed in the entrance region due to large amounts of velocity-slip and temperature-jump. In the fully developed region, the friction coefficient decreases strongly both with increasing *Kn* and aspect ratio but has a weaker dependence on the side angle. The heat transfer coefficient also decreases strongly with increasing rarefaction and aspect ratio; however, as the aspect ratio increases, its sensitivity to *Kn* decreases. Practical engineering correlations are also provided for fully developed flow friction and heat transfer coefficients.

1. Introduction

In recent years, advances in manufacturing technologies and striking developments in micro-fluidic devices have renewed interest in laminar flows through microchannels with irregular cross-sections. Gas flow in a micron-size channel is associated with some degree of non-continuum effect, which is characterized by the Knudsen number, *Kn*, defined as the ratio of the mean-free-path to the appropriate macroscopic flow scale. For the slip-flow regime, which is commonly encountered in microchannel flows, *Kn* is in the range $10^{-3} \leqslant Kn \leqslant 10^{-1}$ and slight rarefaction effects occur adjacent to the walls. In order to account for these rarefaction effects, slip/jump conditions are taken to exist across a thin Knudsen layer, which extends approximately one mean-free-path from the walls. Experimental observations indicate that beyond this layer the continuum hypothesis is valid. Therefore, in the slip-flow regime, it is common practice to use the standard Navier–Stokes and energy equations with only the boundary conditions modified for velocity-slip and temperature-jump at the walls.

For shorter channels, the entrance region is of much importance, where the velocity and temperature fields undergo major transformations from essentially uniform inlet profiles to fully developed profiles. In the entrance region, an analytical solution of the problem is not feasible due to the non-linear inertia terms in the momentum equations. However, the classical boundary-layer approximations provide major simplifications and a number of semi-analytical solutions have been obtained for laminar slip-flow development in the entrance region of circular and parallel-plate channels [1,2]. Clearly, boundary-layer approximations are not suitable for microchannel flows, which are characterized by very low Reynolds numbers.

Gas flows in microchannels have received considerable attention and have been experimentally, numerically and analytically studied by many researchers (e.g. [3–9]). Most of the numerical simulations and analytical solutions were performed in 2D with simplifications in the governing equations. A survey of the available literature on low Mach number flows indicates rather limited information on 3D flows in the slip-flow regime, such as in the entrance region of microchannels. Heat transfer in microchannels has also been studied by a number of researchers mainly in the context of fully developed slip-flows in simpler geometries, which have been recently reviewed [10], and therefore, will not be repeated here. Most of the previous studies as well as the present work consider channels with constant wall temperatures; however, fully developed flows in channels subjected to uniform wall heat fluxes have also been studied (e.g., [11]).

In the literature, studies on thermally developing micro flows are limited to the application of fully developed analytical velocity profiles in the solution of the energy equation for circular tubes and rectangular microchannels. Larrodé et al. [12] obtained an analytical solution for developing heat transfer in a constant wall temperature circular tube, where a fully developed slip velocity profile was employed. The temperature-jump condition was applied at

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Nomenclature

A_c	cross-sectional area
$2a$	top side of the channel (Fig. 1)
c_p	specific heat at constant pressure
D_h	hydraulic diameter, $4A_c/P_{wet}$
f	friction coefficient, $\tau_w/\left(\rho W_i^2/2\right)$
H	channel height
h	heat transfer coefficient
Kn	Knudsen number, λ/D_h
k	thermal conductivity
Nu	Nusselt number, hD_h/k
P_{wet}	wetted perimeter
Pe	Peclet number, $RePr$
Po	Poiseuille number, fRe
Pr	Prandtl number, $c_p\mu/k$
p	pressure
Re	Reynolds number, $\rho W_i D_h/\mu$
T	temperature
u, v, w	velocity components in the x, y, z-directions
$\vec{V}$	velocity vector
W_i	uniform axial inlet velocity
z^+	nondimensional axial position, $z/(D_h Re)$
z^*	reciprocal Graetz number, $z/(D_h RePr)$

Greek symbols

α	aspect ratio, $H/(2a)$
γ	specific heat ratio, c_p/c_v
λ	molecular mean-free-path
μ	dynamic viscosity
ρ	density
τ_w	wall shear stress
ϕ	acute side-angle (Fig. 1)

Subscripts

fd	fully developed
i	inlet
m	mean
S	slip condition
w	wall

the wall in the slip-flow regime. Yu and Ameel [13,14] presented an analytical treatment of the slip-flow problem for rectangular microchannels. Developing and fully developed Nusselt numbers were obtained for hydrodynamically fully developed slip-flow. Note that, in these studies, the effects of axial heat conduction, which becomes important for low Reynolds number conditions, were neglected. Similarly, Lee and Garimella [15] solved the energy equation with a specified velocity profile to study heat transfer in the entrance region of rectangular microchannels subject to uniform wall temperature and uniform wall heat flux boundary conditions. Axial conduction was assumed negligible; moreover, the slip effect was also neglected in the prescribed fully developed velocity profile. Local and average Nusselt numbers were presented for a wide rage of aspect ratios.

Zhen et al. [16] performed a direct Monte Carlo simulation for heat transfer calculations of low speed short microchannel flows. Straight rectangular channels with various aspect ratios were considered. It was concluded that the 2D simplification for a 3D rectangular microchannel flow seems to be reasonable only for aspect ratios smaller than 0.2. Hong and Asako [17] studied the heat transfer characteristics of two dimensional compressible gaseous flows in microchannels and microtubes. Constant wall temperature was considered and a correlation for the prediction of the heat transfer rates was proposed. Important issues associated with microchannel heat transfer were reviewed by Hetsroni et al. [18] including viscous dissipation and axial heat conduction in the fluid as well as channel walls under constant heat flux condition. Maranzana et al. [19] have also investigated the effect of axial conduction in the wall on the heat transfer in microchannels. An axial conduction number defined as the ratio of conductive to convective heat flux was introduced and conditions under which axial conduction is considerable were identified.

Renksizbulut et al. [10] conducted a numerical study of simultaneously developing slip-flow and heat transfer in rectangular microchannels for a wide range of aspect ratios and Knudsen numbers. It was found that the heat transfer and friction coefficients are finite at the channel inlet and greatly reduced in the entrance region due to velocity-slip and temperature-jump at the walls. Poiseuille and Nusselt number correlations were also provided for fully developed friction and heat transfer coefficients, including the effects of axial heat conduction.

The above literature survey indicates a state of incomplete information for simultaneously developing slip-flow and heat transfer in trapezoidal microchannels. In the present work, a wide range of channel aspect ratios ($0.25 \leqslant \alpha \leqslant 2$) and acute side angles ($\phi = 30°, 45°, 60°, 90°$) are considered for $Kn \leqslant 0.1$ in the range $0.1 \leqslant Re \leqslant 10$ with $Pr = 1$. Three-dimensional Navier–Stokes and energy equations with velocity-slip and temperature-jump boundary conditions have been solved numerically by a control-volume method. In particular, the entrance region has been fully investigated, where the major fractions of pressure drop and heat transfer occur in shorter channels that are frequently encountered in modern compact heat exchangers as well as in various types of microfluidic devices found in MEMS.

2. Formulation

Fig. 1 shows the basic geometric variables that describe a trapezoidal channel along with the adopted coordinate system. The top wall of the channel ($2a$) is held constant and the channel height (H) is calculated from the specified aspect ratio defined as $\alpha = H/(2a)$. Thus, a unique cross-section is obtained for a given α and acute side angle ϕ. The channel length is set to a value larger than the estimated hydrodynamic entrance length of the flow to ensure that fully developed conditions are achieved at the exit. Since the entrance length is a function of Re, Kn and geometry, the channel length has been adjusted according to the case under consideration.

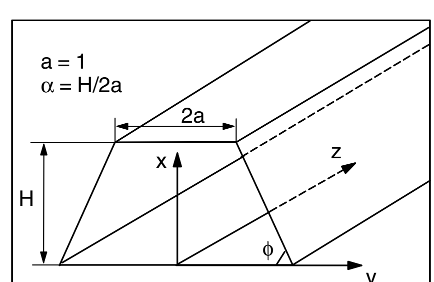

Fig. 1. Channel geometry and the coordinate system.