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New analytical model for heat transfer efficiency of metallic honeycomb structures

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ABSTRACT

In this paper, a new analytical model is presented to investigate the heat transfer performance of sandwich metallic honeycomb structures under the forced convection conditions. The new method overcomes the approximations in the corrugated wall model (also called the modified fin analogy model), the heat transfer efficiency predicted by this new method is consistently lower than that predicted by the corrugated wall model and is higher than that by the effective medium model. Compared with the corrugated wall model and the effective medium model, the new method gives the results closer to the numerical simulation results, which indicates that the method is accurate.

1. Introduction

Honeycombs, 2-D prismatic cellular metals, have emerged as promising multifunctional material systems [1–3]. They have a combination of properties that can be tailored to make them suitable for a range of applications such as structural load support, thermal management, impact energy absorption, sound absorption, fuel cell, energy storage, and others. The properties that appear most attractive are those that govern their use as cores for panels or shells having lower weight than competing materials.

Prismatic structures have one easy flow direction and have attracted significant interest for heat sink application including compact electronic cooling devices and ultra-light actively cooled aerospace structures. Numerous studies have been developed [4–14], thereinto, there are two analytical models used to describe the characteristic of heat transfer in two-dimensional prismatic structures with forced convection. One is the corrugated wall model [4,5] (also called the modified fin analogy model [6,7]). The other is the effective medium model [5]. The effective medium model uses volume averaging technique and somewhat underestimates the heat dissipation because of the assumption that the conduction of heat occurs predominantly normal to the convective flow. The corrugated wall approach can model the detailed cellular structure, thus it was often adopted in latter works. However, this method is also an approximate model.

There are two steps adopted to solve the thermal fields in the corrugated wall model. Firstly, the analysis of heat transfer is performed for the corrugated walls by excluding the effects of fins. Then, the contribution from the fin is added according to the energy balance. Lu said in his paper [4]: except for the effects of fin attachments which are modeled approximately, the approach is felt to be fairly accurate. However, an important aspect is neglected. The contribution of heat loss from fin attachment is about one third of the total heat loss for regular hexagonal honeycomb structures and about the half of the total heat loss for regular rectangular honeycomb structures, but the temperature field of a single corrugated wall is solved by excluding the effects of fins and then is used to decide the heat loss of the corrugated wall. Obviously, the temperature field obtained is not accurate, so that the heat loss of the corrugated wall is also approximate.

In this paper, the transfer matrix method is presented as a new analytical model to analyze the heat transfer for the corrugated walls and fins simultaneously. So it can avoid the approximations in the corrugated model. The article is organized as follows. The model is described in Section 2. Section 3 presents the mathematical formulation of transfer matrix method. The comparisons with other analytical models and the numerical simulation results are discussed and show the validity of method in Section 4 and summary follow.

2. The model

The prototypical compact heat transfer exchanger design is shown in Fig. 1. The cooling fluid, with velocity v_0, temperature T_0, pressure p_0, is forced to flow across a two-dimensional metal array of thickness H sandwiched between two flat rectangular plates of length L and width W. The plates are assumed to be thin and have large thermal conductivity so that the temperature is assumed to be constant along thickness direction. It is insulated on

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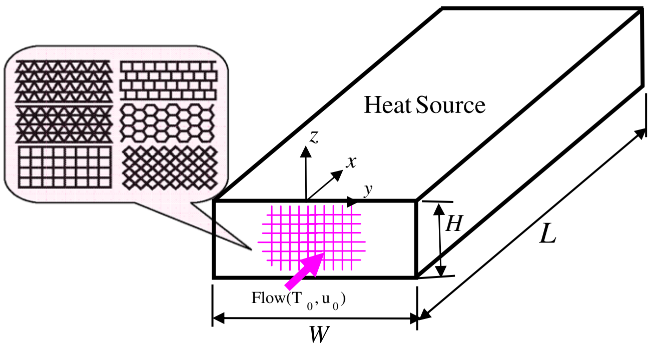

Fig. 1. Prototypical design of compact heat sink with two dimensional metal honeycombs for cooling by forced convection.

the left and right sides and is subjected to the heat sources on the bottom and top surface. The width of channel, W, is assumed to be much larger than the cellular size so that the thermal and hydraulic fields are independent of the coordinate y. Let ρ_f, v_f, μ_f and c_p denote the fluid density, kinematic viscosity, shear viscosity and specific heat constant pressure, respectively. In addition, the usual assumptions of the steady state laminar flow, and constant thermal/physical properties of both fluid and solid are made.

3. Transfer matrix method

Actually, any prismatic structures can be considered here. For the purpose of illustration, the regular hexagonal honeycomb structure (shown in Fig. 2a) is taken as a sample to describe the basic idea of transfer matrix method. Firstly, the heat sink is divided into periodic slices of equal width, as shown in Fig. 2b. Then, one analytical cell is selected along the corrugated wall direction, as shown in Fig. 2c. The variation of temperature T along the cell wall is governed by

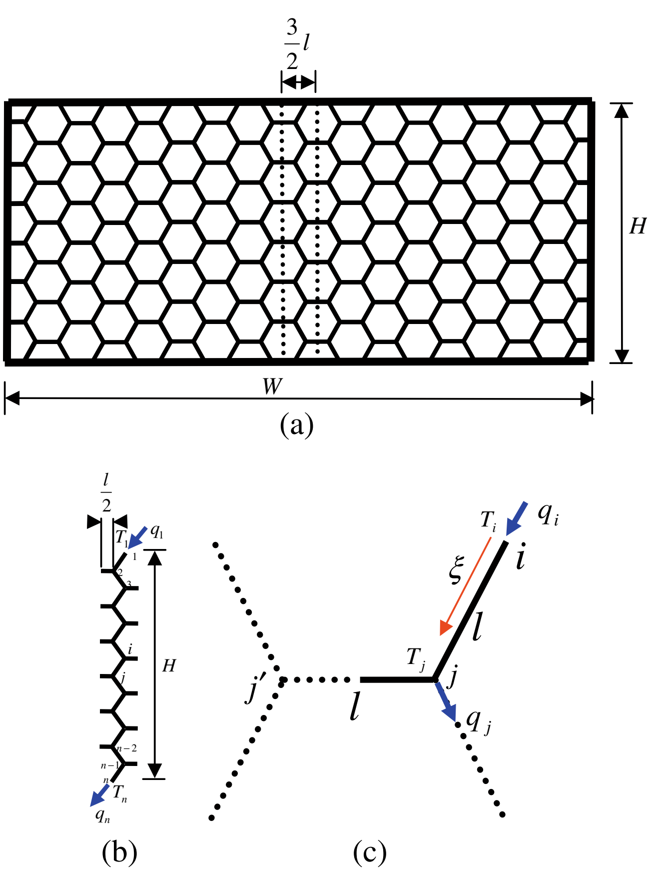

Fig. 2. (a) Cross-section of heat sink for regular hexagonal honeycombs, (b) corrugated wall with the fin attachments and (c) analytical cell (solid line).

$$\frac{\mathrm{d}^2T(x,\xi)}{\mathrm{d}\xi^2}-\frac{2h}{k_st}(T(x,\xi)-T_f(x))=0, \tag{1}$$

where $T_f(x)$ is the mean fluid temperature at the location with coordinate x and ξ is the local coordinate along the wall. k_s is the thermal conductivity ratio of solid wall and t the thickness of the cell wall. h is the local heat transfer coefficient and can be denoted by following expression for regular hexagonal structures

$$h=\frac{Nuk_f}{\sqrt{3}l\sqrt{1-\rho}}, \tag{2}$$

where Nu is Nusselt number, k_f is the thermal conductivity of cooling fluid, ρ is the relative density and l is the cell size.

Assuming

$$\widehat{T}(x,\xi)=T(x,\xi)-T_f(x), \tag{3}$$

$$\lambda^2=\frac{2h}{k_st}. \tag{4}$$

Eq. (1) can be rewritten as

$$\frac{\mathrm{d}^2\,\widehat{T}(x,\xi)}{\mathrm{d}\xi^2}-\lambda^2\,\widehat{T}(x,\xi)=0. \tag{1.1}$$

Subjected the boundary conditions that $\widehat{T}(x,\xi)=\widehat{T}_i$ at $\xi=0$ and $\widehat{T}(x,\xi)=\widehat{T}_j$ at $\xi=l$, the solution of Eq. (1.1) can be expressed as

$$\widehat{T}(x,\xi)=\frac{\sinh[\lambda(l-\xi)]}{\sinh(\lambda l)}\widehat{T}_i+\frac{\sinh(\lambda\xi)}{\sinh(\lambda l)}\widehat{T}_j. \tag{5}$$

Thus, the heat flux along the wall is

$$q(x,\xi)=-k_st\frac{\mathrm{d}T(x,\xi)}{\mathrm{d}\xi}=\lambda k_st\left(\frac{\cosh(\lambda(l-\xi))}{\sinh(\lambda l)}\widehat{T}_i(x)-\frac{\cosh(\lambda\xi)}{\sinh(\lambda l)}\widehat{T}_j(x)\right). \tag{6}$$

The heat dissipation to the fluid through the corrugated wall of length l is

$$q_c=q(x,0)-q(x,l)=\lambda k_st\cdot\tanh(\lambda l/2)(\widehat{T}_i+\widehat{T}_j). \tag{7}$$

The temperature $\hat{T}_{j'}$ at the node j' should be the same with the temperature $\hat{T}_j$ at the node j according to the symmetry, so the heat loss to flow through the fin wall of length $l/2$ is

$$q_f=\frac{1}{2}\lambda k_st\cdot\tanh(\lambda l/2(\widehat{T}_j+\widehat{T}_{j'})=\lambda k_st\cdot\tanh(\lambda l/2)\widehat{T}_j. \tag{8}$$

The energy equilibrium equation can be expressed as below

$$\text{At node}\quad i\ q_i=q(x,0)=\lambda k_st\left(\frac{\cosh(\lambda l)}{\sinh(\lambda l)}\widehat{T}_i-\frac{1}{\sinh(\lambda l)}\widehat{T}_j\right), \tag{9}$$

$$\text{At node}\quad j\quad q_i-q_c-q_f=q_j. \tag{10}$$

Substituting the Eqs. (8) and (9) into the Eq. (10) yields

$$q_j=q_i-\lambda k_st\cdot\tanh(\lambda l/2)(\widehat{T}_i+2\widehat{T}_j). \tag{11}$$

From Eqs. (10) and (11), we can get the expression of q_j, $\widehat{T}_j$

$$q_j=[2\cosh(\lambda l)-1]q_i-\lambda k_st[2\sinh(\lambda l)-\tanh(\lambda l/2)]\widehat{T}_i \tag{12.1}$$

$$\widehat{T}_j=-\frac{\sinh(\lambda l)}{\lambda k_st}q_i+\cosh(\lambda l)\widehat{T}_i. \tag{12.2}$$